**Larbi Ben M'hidi-Oum El Bouaghi University**

**Faculty of Exact Sciences and Natural and Life Sciences**

**Departement of Mathematics and Computer Science**

**First year Licence** **Introduction to probability and descriptive statistics**

## Series N°3 : Combinatorial analysis

**Exercise 01 :** 1) Show that :

$$C_n^1 + C_n^3 + ... = C_n^0 + C_n^2 + ... \quad for\ any\ \ n$$

2) Prove that

$$C_n^1 + 2C_n^2 + ... + nC_n^n = n\,2^{n-1}$$

**Exercise 02 ★ :** 1) For which value of n we have : $7\,P_n^2 = 3\,C_{n+1}^3$.

2) By using the function $x \mapsto (1+x)^n$ calculate $\sum_{k=0}^{n} C_n^k$ , $\sum_{k=0}^{n} (-1)^k\, C_n^k$ *and* $\sum_{k=1}^{n} kC_n^k$.

**Exercise 03 :** Roll 3 different dice randomly and by using the results obtained we construct a 3-digit number.

1. How many numbers can we form ?
2. How many of these numbers are less than 500 and greater than 200 ?
3. How many of these numbers are even ?
4. How many of these numbers have different digits two by two ?

**Exercise 04 :** Passwords have 3 different letters followed by 2 different symbols of the following set $\{@, \%, \$, \star\}$ then 2 numbers. a) How many passwords can you create ?. b) How many of these words start with a vowel and end with an even number ?

**Exercise 05 :** 1) How many different arrangement are there with the letters of the following words : a) Maths b) proposition c) theorem d) arrangement

2) ★ How many different numbers can you write with the digits $3, 3, 3, 5, 5, 1, 2, 2$ ?

**Exercise 06 :** Twenty books are to be arranged on a shelf ; eleven on travel, five on cooking, and four on gardening. How many arrangements are possible :

1. if the books are to be freely grouped.
2. if the books in each category are to be grouped together.
3. if the only books on travel are to be grouped together.

**Exercise 07 :**

1) In a class of 70 students, how many ways can the students be grouped so that there are 12 students in each of the first five groups and ten students in the last one ?.

2) How many ways can the students be grouped so that there are 10 students in each groups ?.

## Series N°4 : Probability

**Exercise 01 :** Express each of the following events by using the events $A, B$ and $C$ :

1. Exactly $A$ occurs.
2. $A$ and $B$ occur.
3. All three events occur.
4. Exactly one of the three events occurs.
5. Exactly two of the three events occurs.
6. None of the three events occurs.
7. At least one of the events occurs.
8. At most one of the events occurs.

**Exercise 02 ★ :** Let $A, B, C$ be three events, such as : $P(A) = 0.3, P(B) = 0.7$, $P(C) = 0.6, P(A \cap B) = 0.2$ and $P(A \cup C) = 0.7$. Find the probability that (1) A or B occur, (2) both A and C occur, (3) A does not occur, (4) at least one of the events occurs, (5) only one event occurs.

**Exercise 03 :**

1. let $(\Omega, \mathcal{F}, P)$ be a probability space.
   i) Show that $\mathcal{G} = \{A \in \mathcal{F}, \quad P(A) = 0 \quad ou \quad P(A) = 1\}$ is a tribe on $\Omega$.
   ii) Let $A \in \mathcal{F}$ be an event. Show that
   $$\mathcal{F}_A = \{A \cap B \ , \quad B \in \mathcal{F}\}$$
   is a tribe on $\Omega$.
2. ★ Let $(\Omega, \mathcal{F}_i)$, $i \in I$, be a probable space. Prove that $\mathcal{F} = \cap_{i \in I} \mathcal{F}_i$ is a tribe on $\Omega$.

**Exercise 04 :** Let $(\Omega, \mathcal{F} = \mathcal{P}(\Omega), P)$ be a probability space.

1) Let $B$ be an event. Show that $P_B$ is a probability on $\Omega$ such as :
$$P_B(A) = \frac{P(A \cap B)}{P(B)} = P(A \mid B).$$

2) ★ Let $a$ be a reel number and $\Omega = \mathbb{R}$. Show that $P_a$ is a probability on $\Omega$ such as :
$$P_a(A) = \delta_a(A) = \begin{cases} 1 & if \quad a \in A \\ 0 & if \quad non \end{cases}$$

**Exercise 05 :** Let $(\Omega, \mathcal{F}, P)$ be a probability space and $A_1, ..., A_n$ $n$ events of $\mathcal{F}$. Prove that :

1) $P\Big(\cup_{i=1}^n A_i\Big) \leq \sum_{i=1}^n P(A_i)$ 2) $P\Big(\cap_{i=1}^n A_i\Big) \geq 1 - \sum_{i=1}^n P(\overline{A_i})$

**Exercise 06 :** A bag contains black and red balls, both marked and unmarked. The probability of observing a marked red ball is 2/10, a marked ball 3/10 and a black ball 7/10.

1. What is the probability of observing a red or marked ball ?
2. What is the probability of observing a unmarked red ball ?
3. What is the probability of observing an unmarked black ball ?
4. What is the probability of observing a black or unmarked ball ?

**Exercise 07 ★ :** In a college, 65% of the students own a car, 82% own a computer, and 55% own both. A student is randomly chosen. What is the probability that the chosen student owns neither a car nor a computer ?

**Exercise 08 :** A box contains $n$ white balls, 5 red balls and 3 green balls. We draw 2 balls randomly.

1. What is the probability that both balls are white ?
2. Let P(n) be the probability that both balls are the same color. Show that $P(n) = \dfrac{n^2 - n + 26}{(n+8) \times (n+7)}$. Calculate $\lim_{n\to\infty} P(n)$, what can you say ?.

**Exercise 09 :** A queue is formed by randomly assigning order numbers to n people. Two friends A and B are in this queue.

1. What is the probability that the two friends are located one behind the other ?
2. What is the probability that the two friends are r places apart (separated by $r - 1$ people) ?

**Exercise 10 :** We roll a 6-sided die. The probability of the face marked i coming up is $p_i$. It is assumed that the probability of each face appearing is proportional to the number written on it.

1. Calculate the probability of each face appearing.
2. Calculate the probability of obtaining an even number.

**Exercise 11 :** There are $n$ students in a class. What is the probability that at least 2 students have the same birthday ? ( Assume that there are 365 days a year).
Calculate this probability if $n = 2$.

**Exercise 12 :** A student prepares for an exam by studying a list of 20 problems. He can solve 11 of them. For the exam, the instructor selects 5 questions at random from the list of 20. Find the probability that the student can solve

1. all five problems on the exam.
2. exactly one problem.
3. at least two problems.
4. at most one problem.