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Faculty of Exact Sciences and Natural and Life Sciences

Departement of Mathematics and Computer Science

First year Licence **Introduction to probability and descriptive statistics**

Answers of the third series : Combinatorial analysis

Answer 01 : 1)Show that :

$$C_n^1 + C_n^3 + ... = C_n^0 + C_n^2 + ... \quad for\ any\ \ n$$

We have

$$(x+y)^n = \sum_{i=0}^{n} C_n^k \, x^{n-k} \, y^k$$

We pose $x = 1$ and $y = -1$, and the previous equation reduces to :

$$0 = \sum_{i=0}^{n} C_n^k \, (-1)^k$$

Wich can be written

$$C_n^0 + C_n^2 + ... = C_n^1 + C_n^3 + ...$$

2) Prove that

$$C_n^1 + 2C_n^2 + ... + nC_n^n = n\, 2^{n-1}$$

This time we begin with the expansion of $(1+x)^n$:

$$(1+x)^n = \sum_{i=0}^{n} C_n^k \, 1^{n-k} \, x^k$$

Differentiating both sides of the previous equation with respect to x gives

$$n\,(1+x)^{n-1} = \sum_{i=1}^{n} C_n^k \, k \, x^{k-1}$$

Now, let $x = 1$, we find :

$$n2^{n-1} = \sum_{i=1}^{n} C_n^k \, k \, x^{k-1} = C_n^1 + 2C_n^2 + ... + nC_n^n$$

Answer 03 : Roll 3 different dice simultaneously and by using the results obtained we construct a 3-digit number.

1. $6 \times 6 \times 6 = 6^3$
2. The number of numbers are less than 500 and greater than 200 is : 3×6^2.
3. 3×6^2.

4. $6 \times 5 \times 4 = P_6^3$

Answer 04 : Passwords have 3 different letters followed by 2 different symbols of the following set $\{@, \%, \$, \star\}$ then 2 numbers.

a) $26 \times 25 \times 24 \times 4 \times 3 \times 10 \times 10 = P_{26}^3 \times P_4^2 \times 10^2$.

b) $6 \times 25 \times 24 \times 4 \times 3 \times 10 \times 5 = 6 \times P_{25}^2 \times P_4^2 \times 10 \times 5$.

Exercise 05 : A box contains n white balls and 5 black balls.

1. We draw 2 balls randomly. The number of ways can we choose
   a) two balls : $C_{n+5}^2 = ...$
   b) two white balls : $C_n^2 = ...$
   c) two balls of the same color : $C_n^2 + C_5^2 = ...$
   d) two balls such that at least one of them is white : $C_n^1 \times C_5^1 + C_n^2 = ...$
2. We draw 2 balls successively with replacement. The number of ways can we choose
   a) two balls : $P_{n+5}^2 = ...$
   b) two white balls : $P_n^2 = ...$
   c) two balls of the same color : $P_n^2 + P_5^2 = ...$
   d) two balls such that at least one of them is white : $P_n^1 \times P_5^1 + P_n^2 = ...$

Answer 06 : 1)a) For the word "Maths"? the number of arrangement is : 5!

b) For the word $''proposition''$ we have $\{p, p, r, o, o, o, s, i, i, t, n\}$ so the number of arrangement is : $\dfrac{11!}{2! \times 3! \times 2!}$.

c) For the word "theorem " we have : $\dfrac{7!}{2!}$.

d) For the word "arrangement" we have : $\dfrac{11!}{2! \times 2! \times 2! \times 2!}$.

Answer 07 : Twenty books are to be arranged on a shelf; eleven on travel, five on cooking, and four on gardening.

1. 20!.
2. We have three groups so 3!. For the books on travel we have 11!, for the books on cooking we have 5! and for the books on gardening we have 4!. So the number of arrangements is : $3! \times 11! \times 5! \times 4!$.
3. $11! \times (5 + 4 + 1)!$ or $11! \times (20 - 11 + 1)!$.

Answer 08 :

1) There are C_{70}^{12} choices for the first group. Having chosen 12 for the first group, there are C_{58}^{12} choices for the second group and so on. The total is :

$$C_{70}^{12} \times C_{58}^{12} \times C_{46}^{12} \times C_{34}^{12} \times C_{22}^{12} \times C_{10}^{10} = \frac{70!}{(12!)^5 \times 10!}$$

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Exercise 09 : We have 30 problems. A student can solve 20 of them. For the exam, the instructor selects 5 questions at random from the list of 30.
The number of ways possible is : $C_{30}^{5} = ...$
The number of ways such that the student can solve

1. all five problems on the exam : C_{20}^{5}.
2. exactly one problem : $C_{20}^{1} \times C_{10}^{4}$.
3. exactly two problems : $C_{20}^{2} \times C_{10}^{3}$.
4. at least two problems :

$$C_{20}^{2} \times C_{10}^{3} + C_{20}^{3} \times C_{10}^{2} + C_{20}^{4} \times C_{10}^{1} + C_{20}^{5}$$

   or

$$C_{30}^{5} - C_{10}^{5} - C_{20}^{1} \times C_{10}^{4}$$

5. at most one problem : $C_{10}^{5} + C_{20}^{1} \times C_{10}^{4}$