Larbi Ben M'hidi-Oum El Bouaghi University

Faculty of Exact Sciences and Natural and Life Sciences

Departement of Mathematics and Computer Science

First year Licence **Introduction to probability and descriptive statistics**

Series N°3 : Combinatorial analysis

Exercise 01 : 1) Show that :

$$C_n^1 + C_n^3 + ... = C_n^0 + C_n^2 + ... \quad for\ any\ \ n$$

2) Prove that

$$C_n^1 + 2C_n^2 + ... + nC_n^n = n\,2^{n-1}$$

Exercise 02 ★ : 1) For which value of n we have : $7\,P_n^2 = 3\,C_{n+1}^3$.

2) By using the function $x \mapsto (1+x)^n$ calculate

$$\sum_{k=0}^{n} C_n^k\,, \quad \sum_{k=0}^{n}(-1)^k\,C_n^k \quad and \quad \sum_{k=1}^{n} kC_n^k$$

Exercise 03 : Roll 3 different dice randomly and by using the results obtained we construct a 3-digit number.

1. How many numbers can we form ?
2. How many of these numbers are less than 500 and greater than 200 ?
3. How many of these numbers are even ?
4. How many of these numbers have different digits two by two ?

Exercise 04 : Passwords have 3 different letters followed by 2 different symbols of the following set $\{@, \%, \$, \star\}$ then 2 numbers.

a) How many passwords can you create ?.

b) How many of these words start with a vowel and end with an even number ?

Exercise 05 : A box contains n white balls and 5 black balls.

1. We draw 2 balls randomly. How many ways can we choose
   a) two balls ?
   b) two white balls ?
   c) two balls of the same color ?
   d) two balls such that at least one of them is white ?

2. We draw 2 balls successively without replacement. Solve the same previous questions.

Exercise 06 : 1) How many different arrangement are there with the letters of the following words : a) Maths b) proposition c) theorem d) arrangement

2) ★ How many different numbers can you write with the digits $3, 3, 3, 5, 5, 1, 2, 2$?

Exercise 07 : Twenty books are to be arranged on a shelf ; eleven on travel, five on cooking, and four on gardening. How many arrangements are possible :

1. if the books are to be freely grouped.
2. if the books in each category are to be grouped together.
3. if the only books on travel are to be grouped together.

Exercise 08 :

1) In a class of 70 students, how many ways can the students be grouped so that there are 12 students in each of the first five groups and ten students in the last one ?.

2) How many ways can the students be grouped so that there are 10 students in each groups ?.

Exercise 09 : A student prepares for an exam by studying a list of 30 problems. He can solve 20 of them. For the exam, the instructor selects 5 questions at random from the list of 30. What is the number of ways possible ?

Find the number of ways such that the student can solve

1. all five problems on the exam.
2. exactly one problem.
3. exactly two problems.
4. at least two problems.
5. at most one problem.