1st year MI Mechanics of material point

Chapter 2: dynamics of the material point

Definition : Dynamics is the study of movements according to the causes that produce them.
This chapter will be devoted to dynamics, the relationships that exist between a movement and the forces that are its cause .

Quantity of movement :

A movement does not depend only on the speed but also on its mass, two different masses which move at the same speed do not arrive in the same way. To do this, we introduce a mathematical quantity which is the quantity of movement.
The momentum relative to the reference frame *R* of a material point *M* , of mass *m* and speed $\vec{V}$ is given by:

$$\vec{p} = m\vec{v}$$

Force vector :

Any force can be represented by a vector (F) whose four properties are:

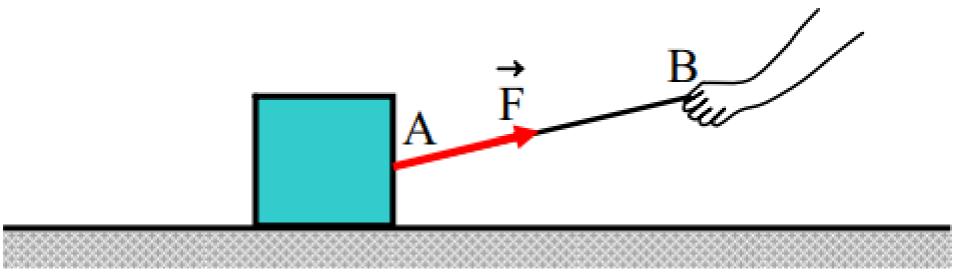
•the direction: straight in which the action is carried out (that of the thread in the Figure).

• the direction: direction in which the action is carried out (from *A* to B , see Figure).

• the point of application: point where the action is exerted on the body (point *A*).

• the module: the intensity of the force with which an appropriate unit is associated.

The forces are additive, that is to say that if N forces act simultaneously on a body, the movement of the latter is the same as in the case where it is subject to the action of a single force equal to the vector sum of N forces. This sum is called the resultant of the N forces.



Force vector

Fundamental law of dynamics:

Principle of inertia :

In a Galilean R frame, the center of inertia of any mechanically isolated material system is either at rest or in uniform rectilinear motion. This is Newton's first law (principle of inertia).

Fundamental principle of dynamics:

Newton's second law: (*it is more of a definition than a law*)

"The derivative of momentum is called force. "

This means that the result of the forces applied to the particle is:

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

This equation is called " **equation of motion** "

Case of constant mass: following what has just been said, if the mass *m* of the mobile is constant (which is common in Newtonian mechanics) then the previous equation is written:

$$\vec{F} = \frac{d(m\vec{v})}{dt} \Rightarrow \vec{F} = m\frac{d\vec{v}}{dt} \Rightarrow \boxed{\vec{F} = m.\vec{a}}$$

Special case: If the resultant *F* is constant then the acceleration *has F* also constant and the movement is **rectilinear uniformly varied.**

This is exactly what happens to bodies that fall in free fall under the effect of force of gravity (called weight):

$$\vec{P} = m.\vec{g}$$

Fundamental Principle of PFD Dynamics

The resultant force acting on a body is equal to the product of its mass times its acceleration.

$$\sum \vec{F} = m\,\vec{a}$$

Example : a body of mass 10 *kg* , subjected to the force *F* = (120 *t* + 40) *N* moves in a straight line. At time *t* = 0 , the body occupies the position $X_0 = 5$ *m* with a speed 1 $V_0 = 6$ *m/s* . Find the speed and position of the mobile as a function of time.

Answer: Using formula (5.3) we find: *F* = (120 *t* + 40) = 10 *a* , such that

a = (*12t* + 4) *ms_2* . –

To find the expression for the instantaneous speed we must integrate the expression for the acceleration.

$$\text{Puisque}\ \frac{dv}{dt} = 12t + 4$$

$$\text{Donc :}\ \int_0^v dv = \int_0^t (12t+4)dt \Rightarrow \boxed{v = 6t^2 + 4t + 6}\left(ms^{-1}\right)$$

Let us integrate again, but this time, the expression obtained for the speed to find the position of the mobile at each moment:

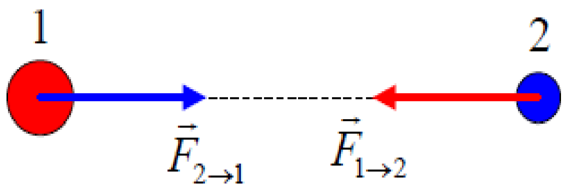
$$\int_0^x dx = \int_0^t v dt = \int_0^t (6t^2 + 4t + 6)dt \Rightarrow \boxed{x = 2t^3 + 2t^2 + 6t + 5}\,(m)$$

Newton's third law or principle of action and reaction:

Statement of the law: " **when two particles are in mutual influence, the force applied by the first particle on the second is equal and of opposite sign to the force applied by the second particle on the first .**

Allows us to write:

$$\boxed{\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}}$$



Action et réaction

Interaction and forces

An interaction between two bodies is manifested by an attraction or repulsion between the bodies considered

Gravitational interaction

It is a force for remote interaction. It is exercised between two masses. It was stated by Newton in 1650.

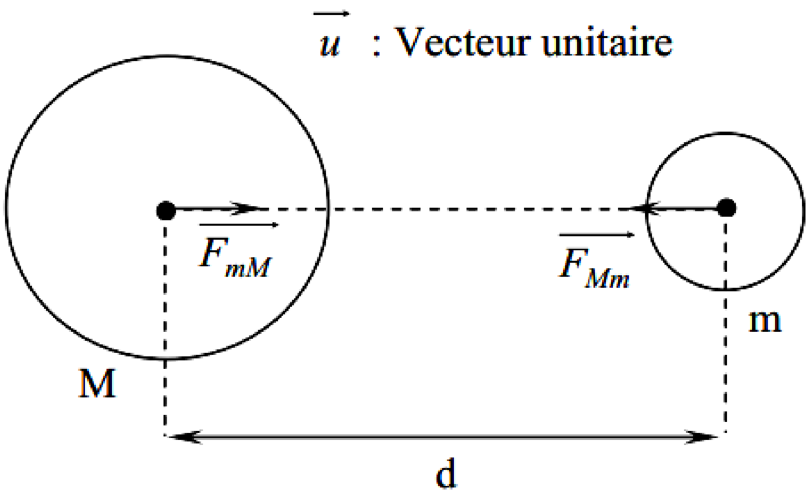
Let a mass M at point O interact with another of mass m at point P, such that the distance between O and P is equal to r, is written:

$$\vec{F} = -G\frac{M.m}{d^2}\vec{u}_r$$

Speed is considered to be the distance traveled per unit of time.

Or $\vec{u}$ is the unit vector oriented from M to m .

G = 6.67210–11 S. I is the constant of universal attraction.



Gravitational interaction

Electromagnetic interaction:

It is represented by electrostatic forces and magnetic forces :

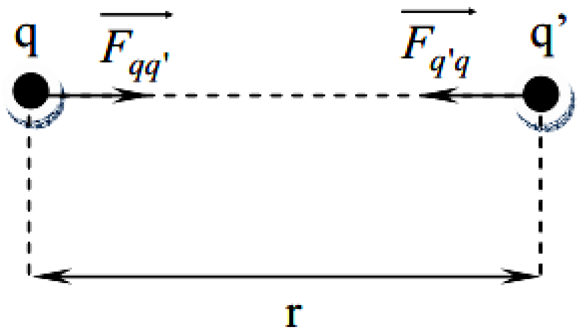
$$\vec{F} = q(\vec{E} + \vec{V} \wedge \vec{B})$$

The electrostatic forces which act between two charges follow Coulomb's law (Figure 3), quite similar to that of gravitation:

$$\vec{F} = K\frac{qq'}{r^2}\vec{u}_r$$

ou $K = \frac{1}{4\pi\mathcal{E}_0} = 9.10^9$ S.I, est la constante universelle électrostatique.

Electrostatic forces are repulsive if the charges are of the same signs or of the same nature, or attractive if they are of opposite signs or of different nature. The magnetic forces of interaction between two currents are also inversely proportional to the square of the distance.

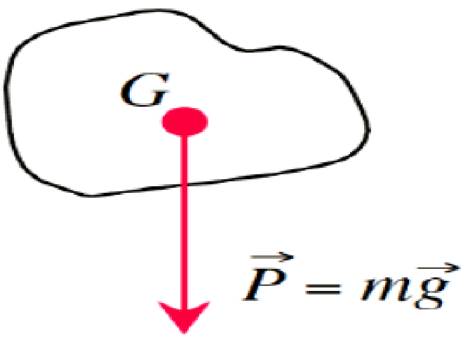


Electromagnetic interaction

Some laws of forces

The law of force : **(or law of mutual influences)** : this law clearly shows the expression of the force (the resultant) applied to a material point in a well-defined situation.

For example : the expression $P = m \cdot \vec{g}$ is the **law of force** which defines the weight of a body in the vicinity of the earth and which allows us to predict the movement of any body in the earth's gravity field.



Poids d'une masse

Having the relation $F = m \, . \, \vec{a}$, we can know the behavior of physical systems, better yet we can even predict their evolution.

FRICTION FORCES :

Each time there is contact between two rough surfaces of two solid bodies, a resistance then appears and opposes the relative movement of the two bodies. There are several types of friction:

_ Friction between solid bodies which can be static and dynamic,

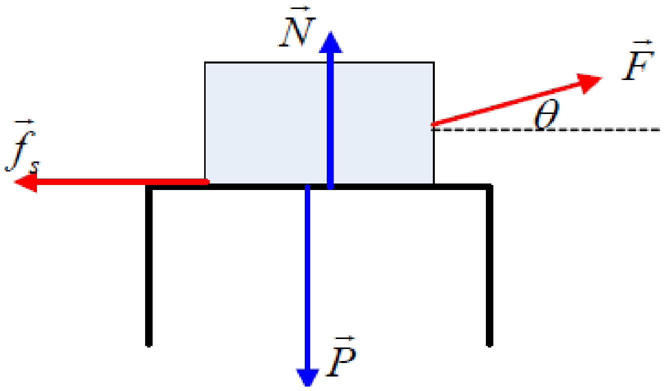
_ Friction in fluids.

Static friction force :

Static friction force is the force that keeps the body in a state of rest even in the presence of an external force.

Case of a body placed on a horizontal plane:

Consider the body of the following figure. It is subject to four forces. Consider $\vec{f}_s$ the static friction force. $\vec{P}$ And $\vec{N}$ are respectively the weight and the reaction force. For the body placed on the table to start moving, a minimum force must be applied $\vec{F}$.



Force de frottement

The body is at rest:

$$\sum_i \vec{F}_i = \vec{0}$$

By projecting onto the two horizontal and vertical axes we obtain:

$$\left.\begin{array}{l} N + F.\sin\theta - P = 0 \\ F.\cos\theta - f_s = 0 \end{array}\right| \Rightarrow \boxed{f_s = F.cos\theta}$$

If the angle θ was zero we would have $f_s = F$ and $P = N$.

Note that $P \neq N$ with $N = P - F .\sin \theta$ which is the force that keeps the body at rest until the force $\vec{F}$ applied manages to tear it from the surface. Just before tearing off the body, the static friction force reaches its maximum value defined by law: $f_s = \mu_s N$ where μ_s is the coefficient of static friction and N is the normal force.

$$N = P - F\sin\theta \Rightarrow \boxed{f_{s,\max} = \mu_s.N = \mu_s(P - F\sin\theta)}$$

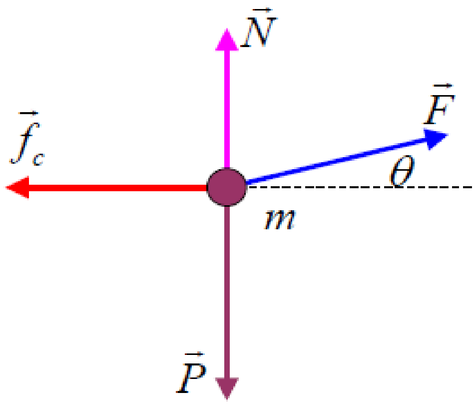
It is necessary that $N > 0$ and therefore $P > F.\sin\ \theta$, otherwise the body rises.

Kinetic friction force:

The force of kinetic friction is the force that opposes the movement of the body on a rough surface. Its intensity is given by the formula:

$$\boxed{f_d = \mu_d N}$$

Note: In the case of **static** friction forces the body is at **rest** , **on the other hand in the case of kinetic or dynamic** friction forces the body is in **motion** .



The body is now considered to be in motion. It is possible to determine the expression for the dynamic friction force after having established the expression for the normal force:

$$\left.\begin{array}{l} N = P - F.\sin\theta \\ f_d = \mu_d N \end{array}\right| \Rightarrow \boxed{f_d = \mu_d(P - F.\sin\theta)}$$

By applying the fundamental relation of dynamics, knowing that m is the mass of the body, we can write:

$$F.\cos\theta - f_d = ma \Rightarrow \boxed{f_d = F.\cos\theta - ma}$$

Where μ_d is the symbol for the coefficient of kinetic (or dynamic) friction and N represents the normal force.

Exercise :

The figure below represents a body whose weight is 5 N and which rests on a rough plane inclined by θ = 35 ° . The coefficient of static friction is 0.80. We take

$g = 10\ m/s$.

a/ What must be the angle of inclination for the body to take off?

b/ What is the maximum static friction force?

c/ What is the normal force for 35 ° ?

d/ What is the static friction force for an inclination of 35 ° ?

solution :

a/ Angle of inclination necessary for the body to take off.

When the static friction force reaches its maximum value for an angle of take-off θ_0, called friction angle and which is a limiting angle, it is balanced with the weight component $\vec{P}_x$, at this moment, the body takes off:

$$\left.\begin{array}{l} f_{s,\max} = P_x = mg\sin\theta_0 \\ f_{s,\max} = \mu_s N \\ N = P_y = mg\cos\theta_0 \end{array}\right| \Rightarrow \boxed{tg\theta_0 = \mu_s} \ ,\ tg\theta_0 = 0,80 \Rightarrow \boxed{\theta_0 = 38,66°}$$

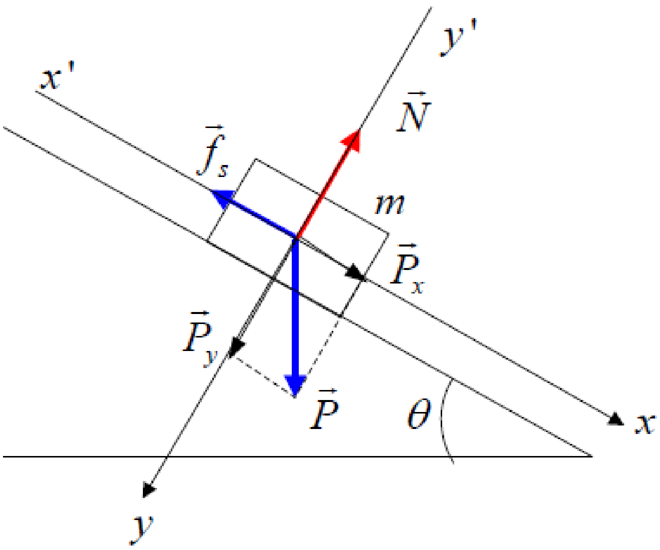
b/ Intensity of the maximum friction force:

$$\boxed{f_{s,\max} = \mu_s N} \ ,\ \boxed{f_{s,\max} = 3,13N}$$

$$\boxed{N = P_y = mg\cos\theta} \ ,\ \boxed{N = 4,1N}$$

d/ Friction force for the 35 ° angle :

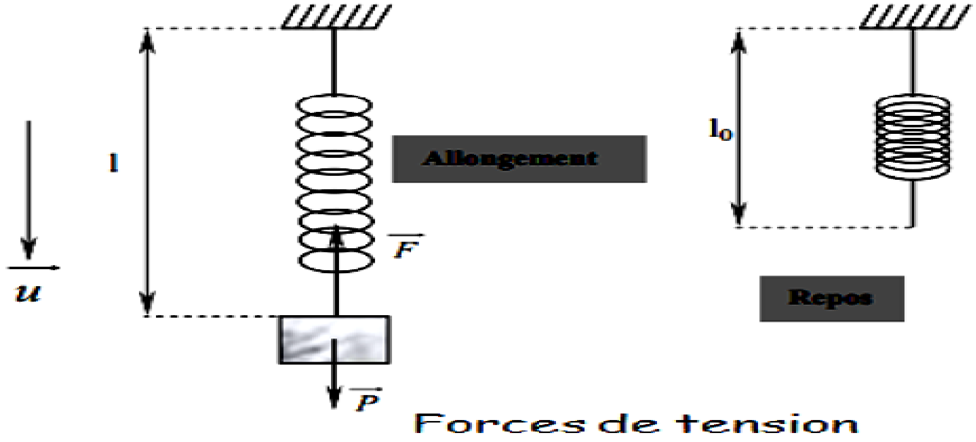
$$\boxed{f_s = P_x = mg\sin\theta} \ ,\ \boxed{f_s = 2,87N}$$



Tension forces

Tension force or restoring force. The simplest example is the spring restoring force.

$$F = -k(l - l_0)$$



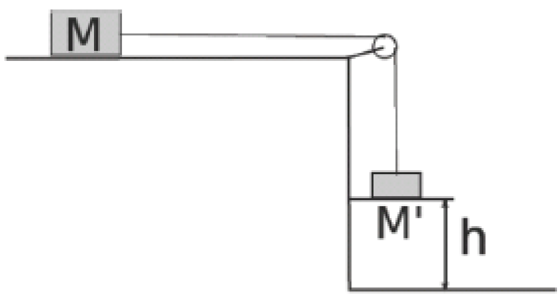
k : elongation coefficient (spring stiffness coefficient).

Application exercise:

Two bodies M and M' of mass m and m' respectively, are connected by an inextensible wire

passing through the throat of a pulley of negligible mass. Initially the body M' is at a height h from the ground, it is released without initial speed. The contact between the body M and the horizontal plane is characterized by static friction coefficients μs and dynamic μd. . μs = 0.6, μd = 0.4, m = 6 kg,

h = 1.5 m and g = 10 m/s²



2- We now take a mass m'= 4 kg, the system starts moving. Considering the two phases of the movement of mass M until it stops:

a- What is the nature of the movement of the mass M. Justify.

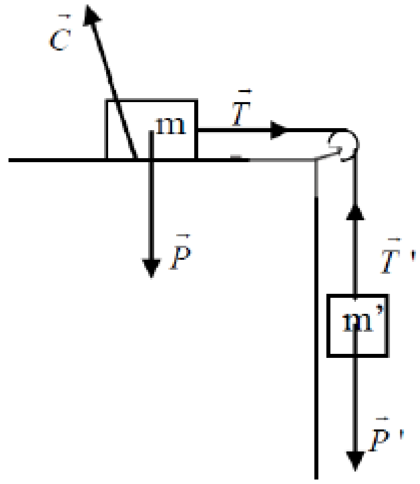
b- Calculate the acceleration in the first phase.

c- Calculate the acceleration in the second phase.

Solution

1- **In balance**

$$\sum \vec{F} = \vec{0}$$



The forces on the mass M':

$$\vec{P'} + \vec{T'} = \vec{0} \qquad\qquad P' - T' = 0$$

Alors $P' = C_x$ et $C_x = \mu_s C_y = \mu_s mg \Rightarrow m' = \mu_s m = 3.6kg$

2- We have two stages

- During the first stage the forces are constant, the acceleration is constant and the speed increases, it is a uniformly accelerated movement.

We apply the fundamental principle of dynamics.

sur la masse M' $\quad \vec{P'} + \vec{T'} = m'\vec{a_1} \Rightarrow P' - T' = m'a_1$

Sur la masse M $\quad \vec{P} + \vec{T} + \vec{C} = m\vec{a_1} \Rightarrow \left[\frac{T - C_x = ma_1}{C_y - P = 0}\right]$

$$P'' - C_x = (m + m')a_1 \Rightarrow a_1 = \frac{(m' - \mu_d m)}{m + m'} g = 1.6\, m/s^2$$

- During the second stage with the presence of friction and the absence of mass M', the speed therefore decreases with a uniformly delayed movement.

$$\vec{P} + \vec{C} = m\vec{a_2} \Rightarrow \left[\frac{-C_x = ma_2}{C_y - P = 0}\right]$$

$$\Rightarrow -\mu_d mg = ma_2 \Rightarrow a_2 = -\mu_d g$$

1st year MI Mechanics of material point

A body of mass **M** is connected to a body of mass **m** = 2 kg via an inextensible wire of negligible mass. A spring **K=150N/m** of negligible mass is attached to the mass **m** and to the wall. 1°)- In the case where we neglect the friction of the mass **m** on the horizontal plane, literally calculate the acceleration taken by the system as well as the tension of the wire. 2°)- Since friction is no longer negligible and the spring is not stretched, what is the maximum value of the mass **M** to be suspended so that the system remains at rest? The value of the coefficient of static friction is μ_s **=0.8** 3°) - We now take a mass M = 3 kg and the spring is stretched by **10cm** , calculate at this position the acceleration of the system and the tension of the wire knowing that the coefficient of dynamic friction is μ_d **=0.25** .