**Larbi Ben M'hidi University** L1

Pr**obability and Statistics TD n°2** 2024-2025

**Exercise 1.** Give $\mathcal{P}(\Omega)$, the set power of $\Omega$ in the following cases

(1) $\Omega = \{1\}$, (2) $\Omega = \{1, 2\}$, (3) $\Omega = \{1, 2, 3\}$, (4) $\Omega = \{1, 2, 3, 4\}$, (5) $\Omega = \varnothing$

**Exercise 2.** Give the simplified expressions of the following sets

(1) $(A \cup B) \cap (A \cup C)$;
(2) $\left(\overline{A} \cup B\right) \cap (A \cup B)$;
(3) $\left(\overline{A} \cup B\right) \cap (A \cup B) \cap \left(\overline{A} \cup \overline{B}\right)$.

**Exercise 3.** $(a)$ How many 8-symbol passwords can you create with 66 characters? $(b)$ If, in a country, cars have plates with 5 characters 2 letters (their alphabet has 26 characters) followed by three numbers, how many possible plates are there?

Repeat the previous questions assuming that repetitions are excluded.

**Exercice 4.** Suppose we dial a random number on my telephone, the number is six digits long. How many possibilities are there. How many possibilities are there if

$(a)$ the number does not contain a 6;
$(b)$ the number contains only even digits;
$(c)$ the number contains the pattern 2345;
$(d)$ the number contains the pattern 2222.

**Exercise 5.** How many different arrangements are there with the letters of the following words: $a)$ pint; $b)$ proposition ; $c)$ Mississippi; $d)$ arrangement ?

**Exercise 6.** We want to form a committee containing 4 of the 23 people in a group. How many of these committees are there?

**Exercise 7.** Prove by calculation that the construction of Pascal's triangle is correct, that is : $C_n^p = C_{n-1}^{p-1} + C_{n-1}^p$.

**Exercise 8.** (1) 21 students must be divided into three groups of 7 people each. The group will be placed in one room and the group in another and the group in a third room. How many possible distributions are there?

(2) we want to distribute 14 computers in two offices of 7 people each. How many are there possible distributions between the two offices if

$a$. the distribution within offices has importance ?
$b$. the distribution within offices has no importance ?

**Exercise 9.** Express each of the following events using events $A, B$ and $C$ and operators union, intersection and complementation :

(1) At least one of the events occurs;
(2) At most one of the events occurs;
(3) None of the three events occurs;
(4) All three events occur;
(5) Exactly one of the three events occurs;
(6) $A$ and $B$ occur, but $C$ does not occur;
(7) $A$ occurs, otherwise $B$ does not occur no more.

**Exercise 10.** Let $A$ and $B$ be two events such that : $P(A) = 1/4$, $P(B) = 2/3$ and $P(A \cap B) = 1/8$. Calculate the probability of

$$E = \{\text{at least one event occurs}\}$$
$$F = \{\text{only one event occurs}\}$$

**Exercise 11.** An urn contains 7 red and 5 blue balls, which we take out (without looking!), one by one.

$(a)$ What is the probability that the first ball is blue ?

$(b)$ What is the probability that the last ball is blue ?

$(c)$ What is the probability that the last ball is blue, given that the first ball is blue.

**Exercise 12.** We consider a stock of electric ampoules of which 70% come from factory 1 and 30% from factory 2. There are two types of ampoules: type $A$ and type $B$. Factory 1 produces 80% of ampoules type $B$ and factory 2 produces 60%. An ampoule is taken at random from the stock.

(1) What is the probability that the ampoule is type $B$ ?

(2) What is the probability that it is from factory 1 given that it is type $B$ ?

**Exercise 13.** Let $\Omega = \{1, 2, 3, 4, 5, 6\}$. We define on $\Omega$ two probabilities $P$ and $P'$ such that :

$P(1) = 3/10$, $P(2) = 1/5$, $P(3) = 1/20$, $P(4) = 3/20$, $P(5) = 1/20$, $P(6) = 1/4$. and for all $i \in \{1, 2, \ldots, 6\}$, $P'(i) = 1/6$.

We consider the events $A = \{1, 2, 5, 6\}$, $B = \{2, 3\}$.

Study the independence of $A$ and $B$ relative to $P$ and relative to $P'$.