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Chapter 3: Linear Programming: Graphical Method

To solve linear programming problems, several different methods are used, following successive steps and starting from the initial solution to the optimal solution. The most important of these methods is graphical method.

The graphical method

Any LP with only two variables can be solved graphically. We always label the variables x_1 and x_2 and the coordinate axes the x_1 and x_2 axes. The steps of graphical method for solving an LPP are as follows:

1. Plot the graph corresponding to the given constraints.
2. Determine the region for each given constraint.
3. Determine the feasible region.
4. Determine corner/extreme points.
5. Examine corner/extreme points.1.

1. The graphical method in the case of maximization:

The following example explains steps of graphical method in the case of maximization.

Max: $Z=8X_1+6X_2$

$4X_1+2X_2\leq 60$

$2X_1+4X_2\leq 48$

$X_1\geq 0,\ X_2\geq 0$

Solution:

a. Draw the coordinate axes corresponding to the variables of the problem and the graphical representation of all constraints

- Converting the inequalities forming the constraints of the problem into equations as follows:

$$4X_1+2X_2= 60$$

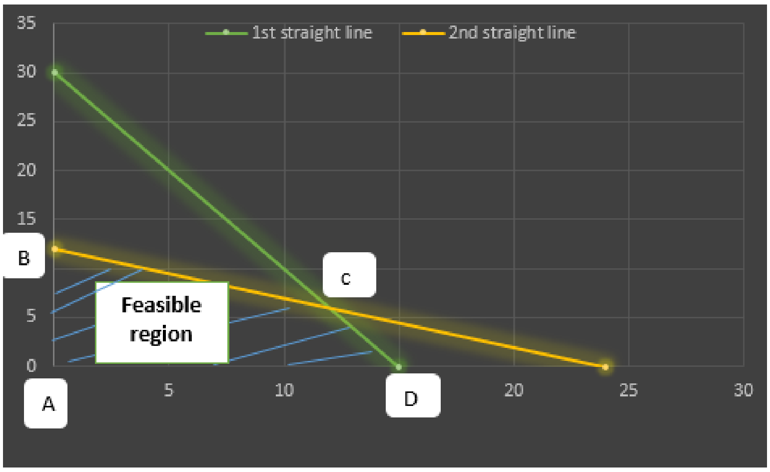
$$2X_1+4X_2= 48$$

- To draw the two straight lines, it is enough to find two points for each line as follows:

1st straight line: $4X_1+2X_2= 60$			2nd straight line 2: $2X_1+4X_2= 48$		
	X_1	**X_2**		**X_1**	**X_2**
point 1	0	30	point 1	0	12
point 2	15	0	point 2	24	0
	After drawing the line, we shade the region that achieves the inequality solutions for the first constraint, which are all the points under the straight line			After drawing the line, we shade the region that achieves the inequality solutions for the second constraint, which are all the points under the straight line	
	The **feasible region** is the area common to the two lines				

	and includes the points (A B C D), which allows us to determine the optimal solution, which is point C

The **feasible region** is the shaded area in the polygon represented by the points (A B C D), and therefore any point within this area represents a possible solution to the problem. The best solution is represented by the three points B, C, and D (except for the point of principle A, which represents the point of origin). The best solution is one of these four points.



C. Determining the optimal solution: Determining the optimal solution from among the four corner points is done in one of the following ways:

- The method of evaluating all corner points;

- Method of drawing the objective function:

C.1 Evaluate the corner points:

Since c is the point of intersection of the two lines, we solve the sentence of the two equations as follows:

$$\begin{cases} 4x_1+2x_2= 60 \\ 2x_1+4x_2=48 \end{cases}$$

$$\begin{cases} 2x_1+x_2= 30 \\ x_1+2x_2=24 \end{cases} \qquad x_1=24-2x_2$$

$$2(24-2x_2)+x_2= 30 \qquad 48-4\ x_2+\ x_2= 30$$

$$\mathbf{X_1= 12,\ x_2=6}$$

To confirm this, we make up in the mathematical model:

- **point A (0, 0):** Z= 0, 0<60, 0<48, (0, 0) = (0, 0)

- **point B (0, 12):** Z= 72, 24<60, 48≤48, (0, 0)≥ (0, 12)

- **point D (15, 0):** Z= 120, 60≤60, 30<48, (0, 15)≥ (0, 0)

-point C (12, 6): Z= 132, 60=60, 48=48, (12, 6)≥ (0, 0)

Feasible solutions to the LP:

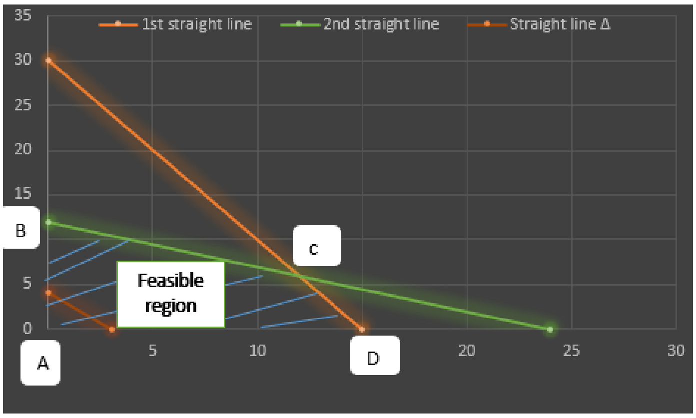
corner points			Z
	X_1	X_2	
A	0	0	0
B	0	12	72
C	12	6	132
D	15	0	120

Through these possible solutions, we notice that the value of the objective function Z rose from zero to the value 132, which is the largest value, that is, at the point C, it achieves the optimal solution. In other words, the optimal production plan is to produce 12 unit of X1 and 6 unit of X_2 to achieve the optimum value 132 unit under the production capacities available.

C.2 draw the objective function

The same solution reached in the previous method can be reached in an easier way by drawing the objective function. To create the line representing the objective function, the objective function can be given any value, let it be 24 units, that is:

Straight line Δ: $Z=8X_1+6X_2= 24$		
	X_1	**X_2**
point 1	0	4
point 2	3	0
The straight line passing through the binary (3, 4) is the carrying line of the objective function when it is equal to 24. We move this line parallel to the formed vertices of the polygon of possible solutions, where it seems from the drawing that the point C is the ideal point because it is the last point reached by the line Δ		



2. The data method in case of minimizing:

Solve the following linear program using the graphical method:

$$\text{Min: } Z=5X_1+5X_2$$

S / T
$$\begin{cases} 3X_1+X_2\geq 6 \\ X_1+2X_2\geq 6 \\ X_1\geq 0,\ X_2\geq 0 \end{cases}$$

a. Draw the coordinate axes corresponding to the variables of the problem and the graphical representation of all constraints

- Converting the inequalities forming the constraints of the problem into equations as follows:

$$3X_1+X_2= 6$$

$$X_1+2X_2= 6$$

- To draw the two straight lines, it is enough to find two points for each line as follows:

1st straight line : $3X_1+X_2= 6$			**2nd straight line: $X_1+2X_2= 6$**		
	X_1	**X_2**		**X_1**	**X_2**
point 1	0	6	point 1	0	3
point 2	2	0	point 2	6	0
	After drawing the line, we shade the region that achieves the inequality solutions for the first constraint, which are all the points above the straight line.			After drawing the line, we shade the region that achieves the inequality solutions for the second constraint, which are all the points above the straight line	
	The **feasible region** is the area common to the two lines and includes the points (B C D), which allows us to determine the optimal solution, which is point C.				

B.1 Evaluate the corner points:

Since c is the point of intersection of the two lines, we solve the sentence of the two equations as follows:

$$\begin{cases} 3x_1+x_2= 6 \\ x_1+2x_2=6 \end{cases}$$

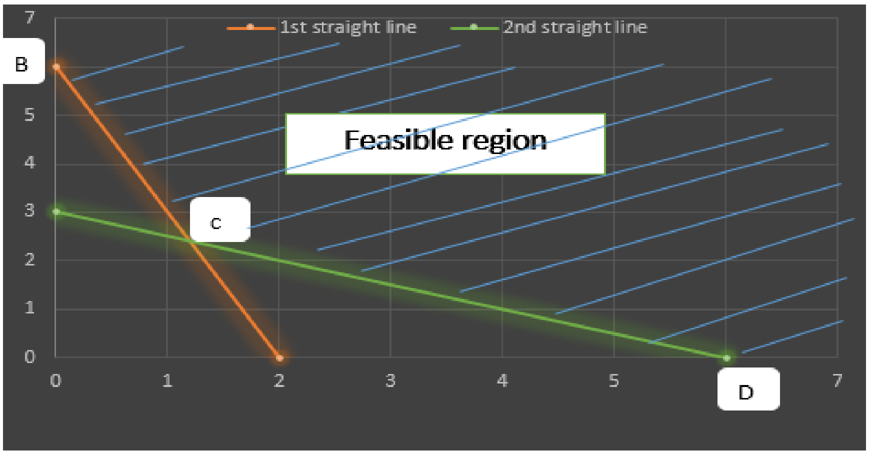
$$x_1=6-2x_2$$

$$3(6-2x_2)+x_2= 6 \qquad 18-6\,x_2+x_2= 6$$

$$\mathbf{X_2= 12/5= 2.4,\ X_1=1.2}$$

Feasible solutions to the LP:

Corner points			Z
	X_1	X_2	
B	0	6	30
C	1.2	2.4	18
D	6	0	30

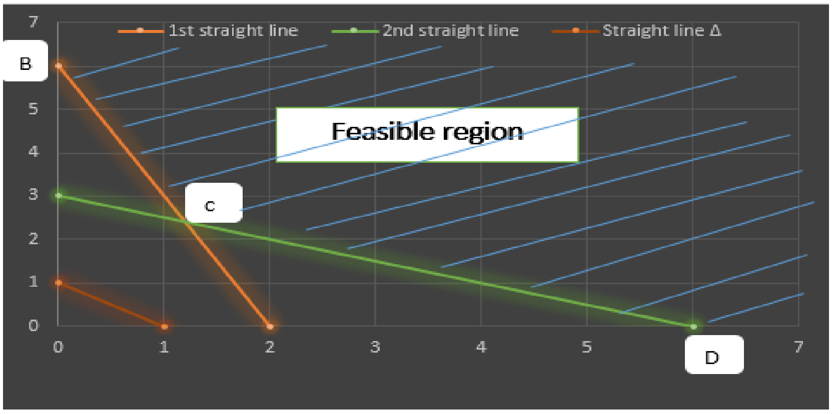


Through these possible solutions, we note that the smallest value of the objective function Z is 18, which was achieved at the point of intersection C, which represents the optimal solution.

B.2 draw the objective function

To create the line representing the objective function, the objective function can be given any value, let it be 5, that is:

straight line Δ**:** $Z=5X_1+5X_2=5$		
	$\mathbf{X_1}$	$\mathbf{X_2}$
point 1	0	1
point 2	1	0
The straight line passing through the binary (1, 1) is the carrying line of the objective function when it is equal to 5. We move this line parallel to the formed vertices of the polygon of possible solutions, where it seems from the drawing that the point C is the optimal point because it is the first point reached by the line Δ		


1st straight line
2nd straight line
Straight line Δ
B
C
D
Feasible region
0
1
2
3
4
5
7