University of Oum El Bouaghi

Faculty of Economic Sciences, Commercial Sciences and Management Sciences

First year, common trunk

Academic year : 2023-2024

Mathematics 1 Module
Solution of Series 03 (Numerical functions).

Exercise 01: Domain of the functions

$$D_f = IR - \{-3,1\}, (c \quad , \quad D_f = \left[\frac{-1}{2}, +\infty\right[\ (b, \qquad D_f = IR - 4 \ (a$$

$$D_f = [1, +\infty[\ (e, \qquad D_f =]-\infty, 3[(d$$

Exercise 02: The limits

1)

$$\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$$

2)

$$\lim_{x\to +0} \left(\sqrt{\frac{1}{x^2}+\frac{1}{x}+1} - \sqrt{\frac{1}{x^2}+\frac{1}{x}-1}\right) =$$

$$= \lim_{x\to +0} \frac{\left(\sqrt{\frac{1}{x^2}+\frac{1}{x}+1} - \sqrt{\frac{1}{x^2}+\frac{1}{x}-1}\right)\left(\sqrt{\frac{1}{x^2}+\frac{1}{x}+1} + \sqrt{\frac{1}{x^2}+\frac{1}{x}-1}\right)}{\left(\sqrt{\frac{1}{x^2}+\frac{1}{x}+1} + \sqrt{\frac{1}{x^2}+\frac{1}{x}-1}\right)}$$

$$= \lim_{x\to +0} \frac{2}{\left(\sqrt{\frac{1}{x^2}+\frac{1}{x}+1} + \sqrt{\frac{1}{x^2}+\frac{1}{x}-1}\right)} = 0,$$

3)

$$\lim_{x\to +\infty} \frac{4x^4-2x^3+6}{2x^4+2x^2+3} = \lim_{x\to +\infty} \frac{4-2/x+6/x^4}{2+2/x^2+3/x^4} = 2.$$

Exercise 03:

1) We have f is continuous for $x \neq 5$, because it is rational function. At $x = 5$ We obtain : $\lim_{x\to 5} f(x) = f(5)$, then f is continuous on IR.

2) We have g is continuous for $x \neq 2$, because it is rational function. At $x = 2$ We obtain :

$$\lim_{x\to 2} g(x) = \lim_{x\to 2}\frac{1}{\sqrt{x-1}+1} = \frac{1}{2}$$

Then g is continuous if : $2b+1=\frac{1}{2} \to b=\frac{-1}{4}$.

Exercise 04:

1) $S=\left\{\frac{-1}{3}\right\}$, 2) $S=\emptyset$, 3) $S=\{ln(4)\}$, 4) $S=\{ln(3)\}$, 5) $S=\left\{e^{\frac{1}{2}}\right\}$.

Exercise 05:

1) **The domain of equation is**

$$\begin{cases} x-1>0 \\ x-3>0 \end{cases} \Leftrightarrow \begin{cases} x>1 \\ x>3 \end{cases} \Leftrightarrow \begin{cases} x\in]1;+\infty[\\ x\in]3;+\infty[\end{cases} \Leftrightarrow x\in]1;+\infty[\cap]3;+\infty[=]3;+\infty[$$

Then, we have :

$$\ln(x-1)+\ln(x-3)=\ln(3) \Leftrightarrow \ln((x-1)(x-3))=\ln(3) \ (\text{car } \ln(a)+\ln(b)=\ln(a\times b))$$

$$\Leftrightarrow (x-1)(x-3)=3 \Leftrightarrow x^2-4x=0 \Leftrightarrow x(x-4)=0 \Leftrightarrow x=0 \text{ ou } x=4.$$

Finally $S=\{4\}$.

2) **The equation is defining if**

$$x\in]0;+\infty[$$

Then

$$\ln x=2 \Leftrightarrow \ln x=2\times 1 \Leftrightarrow \ln x=2\times \ln e \Leftrightarrow \ln x=\ln(e^2) \Leftrightarrow x=e^2. \quad \boxed{S=\{e^2\}}$$

3) **The domain of equation is $]0,+\infty[$. We put $X=ln(x), then$**

$X^2+X-6=0$, **with two solution**

$X=2 \Leftrightarrow \ln x=2 \Leftrightarrow x=e^2$

$X=-3 \Leftrightarrow \ln x=-3 \Leftrightarrow x=e^{-3}$.

$$\boxed{S=\{e^2;e^{-3}\}}$$

4) **The equation is defining if**

$$x\in \left]-\infty;\frac{1}{2}\right[\cup \left]\frac{1}{2};1\right[\cup]1;+\infty[.$$

Then

$$\ln\left(\left|\frac{x-1}{2x-1}\right|\right)=0 \Leftrightarrow \left|\frac{x-1}{2x-1}\right|=1 \Leftrightarrow \frac{x-1}{2x-1}=1 \text{ ou } \frac{x-1}{2x-1}=-1.$$

Finally

$$\boxed{S=\left\{0;\frac{2}{3}\right\}}$$

Exercise 06:

1) **L'inequlitie is defining if**

$$2x-5>0 \Leftrightarrow x\in \left]\frac{5}{2};+\infty\right[$$

then

$$\ln(2x-5)\geq 1 \Leftrightarrow \ln(2x-5)\geq \ln(e) \Leftrightarrow 2x-5\geq e \Leftrightarrow x\geq \frac{e+5}{2}$$

$$S=\left]\frac{5}{2};+\infty\right[\cap\left]\frac{e+5}{2};+\infty\right[=\boxed{\left]\frac{e+5}{2};+\infty\right[},$$

2) **We have**

$$S=\left]\frac{-1}{2},1\right]$$

3) **We put**

$X = \ln(x), \textit{then}$

$$X^2-3X+2\geq 0$$

then

$$S=]-\infty,0]\cup[\ln 2,+\infty[,$$

Exercise 07:

1) **The system is defining if** $x>0 \text{ and } y>0$, **then :**

$$\begin{cases} x-y=\frac{3}{2} & L_1 \\ \ln x+\ln y=0 & L_2 \end{cases} \Leftrightarrow \begin{cases} y=x-\frac{3}{2} & L_1 \\ \ln x+\ln\left(x-\frac{3}{2}\right)=0 & L_2 \end{cases} \Leftrightarrow \begin{cases} y=x-\frac{3}{2} & L_1 \\ \ln\left[x\left(x-\frac{3}{2}\right)\right]=0 & L_2 \end{cases} \Leftrightarrow \begin{cases} y=x-\frac{3}{2} & L_1 \\ x\left(x-\frac{3}{2}\right)=1 & L_2 \end{cases}$$

We have :

$$x^2-\frac{3}{2}x-1=0 \Leftrightarrow 2x^2-3x-2=0$$

Finally

$$S=\left\{\left(2;\frac{1}{2}\right)\right\}$$

2) The system is defining if $x > 0$ *and* $y > 0$.
We put $X = ln(x), Y = ln(y)$, then

$$\begin{cases} 5X+2Y=26 & L_1 \\ 2X-3Y=-1 & L_2 \end{cases} \Leftrightarrow \begin{cases} 15X+6Y=78 & 3L_1 \\ 4X-6Y=-2 & 2L_2 \end{cases} \Leftrightarrow \begin{cases} 19X=76 & 3L_1-2L_2 \\ Y=\dfrac{2X+1}{3} & L_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} X=\dfrac{76}{19}=4 & 3L_1-2L_2 \\ Y=\dfrac{2\times4+1}{3}=3 & L_2 \end{cases} \boxed{\Leftrightarrow \begin{cases} X=4 & 3L_1-2L_2 \\ Y=3 & L_2 \end{cases}}$$

$$X=4 \Leftrightarrow \ln x=4 \Leftrightarrow x=e^4 \text{ et } Y=3 \Leftrightarrow \ln y=3 \Leftrightarrow y=e^3$$

$$S=\left\{\left(e^4;e^3\right)\right\}$$

3) We have

$$\begin{cases} e^x+2e^y=3 \\ x=-y \end{cases} \to e^{-y}+2e^y=3 \to 2e^{2y}-3e^y+1=0 \to 2Y^2-3Y+1=0$$

$$Y_1=1, Y_2=\frac{1}{2} \to y_1=ln1, y_2=ln\frac{1}{2}$$

$$x_1=-ln1, x_2=-ln\frac{1}{2}$$