University of Oum El Bouaghi

Faculty of Economic Sciences, Commercial Sciences and Management Sciences

First year, common trunk

Academic year : 2023-2024

Mathematics 1 Module
Solving the second series (Numerical sequences)

Exercise01:

Study of monotonicity and convergence

$$1)\ u_{n+1}-u_n=(n+1)^2+3-n^2-3$$
$$=2n+1>0,\quad \forall n\in IN.$$
$$lim\ u_n=+\infty$$

The sequence is completely increasing and diverging.

$$2)\ u_{n+1}-u_n=-4(5)^{n+1}+4(5)^n=-16(5)^n<0$$
$$lim\ u_n=+\infty.$$

The sequence is completely diminished and divergent.

Exercise02:

I)

$$1)\ u_0+20r-u_0-10r=25\rightarrow r=\frac{5}{2}$$

$$2)\ u_7=u_0+7r=37\ \text{ and }\ u_3=u_0+3r=13$$

$$\rightarrow u_0=-5, r=6, S_8=(u_0+u_8)\frac{9}{2}=(2u_0+8r)\frac{9}{2}$$

II)

$$(u_3+u_n)\frac{n-2}{2}=6456\rightarrow(2u_0+3r+nr)\frac{n-2}{2}=6456$$

$$\rightarrow(19+5n)\frac{n-2}{2}=6456\rightarrow 5n^2+10n-38=2*6456$$

Exercise03:

1) First, we have

$$U_1=U_0+6000*8\%=6480$$

And since the increase is constant,

$$\boxed{U_2 = 6960}$$

$$U_3 = 6960 + 6000 \times \frac{8}{100}$$

$$= 6960 + 480$$

$$\boxed{U_3 = 7440}$$

2) Since the increase is constant, we conclude that : $U_{n+1} - U_n = 480.$

Hence, the value of the amount each year is arithmetic successive terms, where the general term is

$$U_n = U_0 + nr = 6000 + 480n$$

The number of years that must be waited for the initial amount to double to 3 times :

$$U_n = 3 * 6000 = 18000$$

$$18000 = 6000 + 480n \rightarrow n = 25$$

Exercise04:

1) $u_2 = 4$, $u_1 = 2$, $u_0 = 1$.

2) *Since* : $u_{n+1} = 2\,u_n$, $\forall n \in IN$, **we conclude that** (u_n)

is a geometric sequence whose base is 2.

3) $u_0 + u_1 + \cdots + u_n = u_0 \frac{1-q^{n+1}}{1-q}$.

4) Since $q = 2 > 1$, we have $(u_n)_{n \in IN}$, **is divergent.**

Exercise05

1) Since

$$w_{n+1} = v_{n+1} - u_{n+1} = \frac{u_n + 2v_n}{3} - \frac{2u_n + v_n}{3} = \frac{1}{3} w_n$$

we conclude that (w_n) **is a geometric sequence whose base and general term are**

$$q = \frac{1}{3}, \qquad w_n = \left(\frac{1}{3}\right)^n.$$

2) We show that the two sequences are adjacent.

$$u_{n+1} - u_n = w_n, \quad v_{n+1} - v_n = -w_n$$

Then (u_n) **is** $\uparrow$ **, and** (v_n) **is** $\downarrow$ **, and lim(**$(u_n) - (v_n)$ **)=0.**

Exercise06:

$u_1 = u_0 + 0.06\, u_0 = 11000 * 1.06$

and

$u_2 = u_1 + 0.06\, u_1 = u_1 * 1.06$ **: 2002**

$u_3 = u_2 + 0.06\, u_2 = u_2 * 1.06$ **: 2003**

2) From the definition of complex interest, we are concluded

$u_{n+1} = u_n + 0.06\, u_n$

3) We conclude that (u_n) is a geometric sequence.

Exercise07:

1)

$u_2 = \frac{26}{9}, u_1 = \frac{8}{3}, u_0 = 2$

2) Backward proof

First :

$$u_0 = 2$$

Then, we have :

$$u_{n+1} \leq 3 \leftarrow \forall n \in IN, u_n \leq 3$$

3) The attached function of the sequence (u_n) is

$$f(x) = \frac{x}{3} + 2$$

It is an increasing function, then (u_n) is a monotonous sequence, and we have

$$, u_1 \geq u_0 = 2$$

We conclude that (u_n) is increasing.

4) From the previous answers, the sequence (u_n) is increasing and limited from above, and from there it is convergent.