University of Oum El Bouaghi
Faculty of Economic Sciences, Commercial Sciences and Management Sciences
First year, common trunk

Mathematics 1 Module

Chapter 01 : Combinatorial Analysis.

Introduction

This chapter deals with finding effective methods for counting the number of ways that things can occur. In fact, many problems in probability theory can be solved simply by counting the number of different ways that a certain event can occur. The mathematical theory of counting is formally known as combinatorial analysis.

1) Factorial (!)

Definition: Let n be a natural number. It is represented as $n!$ and it is read as n the multiplication factor.

$$n! = n \cdot (n-1) \cdot (n-2) \ldots \ldots 3 \cdot 2 \cdot 1$$

Property: From the definition, it is concluded that:

$$n! = n \cdot (n-1)!$$

Examples

- $3! = 3 \cdot 2 \cdot 1 = 6.$
- $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$
- $6! = 6.\ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6.\ 5! = 720.$
- Simplify the following relationships :

$$\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1), \forall n$$

2. Arrangements without repetition :

Definition: Let E be a set with n element, and p is a natural number where :

$$p \leq n.$$

Every sequenced subset of $\boldsymbol{E}$ includes $\boldsymbol{p}$ element is called Arrangements without repetition of $\boldsymbol{p}$ element from **E**. It is written as follows:

$$\boldsymbol{A_n^p} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{p})!}$$

Example:

Let : **E = {a,b,c}.**

The arrangements of two elements from $\boldsymbol{E}$ are the sequenced subsets of two elements from $\boldsymbol{E}$, and they are:

{a, b}, {b, a}, {a, c}, {c, a}, {b, c}, {c, b} = **6 sets.**

In another way,

$$\boldsymbol{A_3^2} = \frac{\mathbf{3}!}{(\mathbf{3}-\mathbf{2})!} = \mathbf{6}.$$

Special case

If $\boldsymbol{p} = \boldsymbol{n}$, $\boldsymbol{A_n^n} = \boldsymbol{n}!$ is obtained. It is called permutations without repetition, and it is written as follows:

$$\boldsymbol{P_n} = \boldsymbol{n}!$$

Example:

Let : **E = {a,b,c}.**

The permutations of three elements from $\boldsymbol{E}$ are the sequenced subsets of three elements from $\boldsymbol{E}$, and they are:

$$A_3^3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 6.$$

3) Arrangements with repetition

Every sequenced subset from **E** includes $\boldsymbol{p}$ element where each element can be repeated is called Arrangements with repetition of $\boldsymbol{p}$ from **E**. In this case the following is obtained:

$$A_n^p = n^p, 1 \leq p \leq n$$

Example:

Find all the numbers composed of three digits from the following set :

9, 7, 5, 3, 1.

The numbers composed of three digits are:

$$A_5^3 = 5^3 = 125.$$

4.Combinations without repetition

E is a set of $\boldsymbol{n}$ element and $\boldsymbol{p}$ is a natural number where : $\boldsymbol{p} \leq \boldsymbol{n}$.

Every subset of **E** that includes $\boldsymbol{p}$ element is called a combination of $\boldsymbol{p}$ element from **E**.

The number of combinations is as follows:

$$C_n^p = \frac{n!}{p!(n-p)!}$$

Example

Let : **E= {a,b,c}.**

The Combinations of two elements from **E** are the subsets of two elements from **E**, and they are:

{a, b}, {a, c}, {c, b} = 6 sets

In another way :

$$C_3^2 = \frac{3!}{2!\,(3-2)!} = 3.$$

Properties

Let $\boldsymbol{k}$ and $\boldsymbol{n}$ be two natural numbers, we have :

1) For all $n \in IN$:

$$C_n^k = C_{n-1}^k + C_{n-1}^{k-1}, \quad \forall 1 \leq k \leq n-1$$

2) For all $n \in IN$:

$$C_n^k = C_n^{n-k}, \qquad \forall k \leq n$$

3)For all $n \in IN$:

$$C_n^n = C_n^0 = 1$$

<u>4. Newton's Binomial Theorem</u>

For all $a, b \in IR$, For all $n \in IN^*$:

$$(a+b)^n = C_n^0\, a^n b^0 + C_n^1\, a^{n-1} b^1 + C_n^2\, a^{n-2} b^2 + \ldots\ldots + C_n^n\, a^0 b^n$$
$$= \sum_{p=0}^{n} C_n^p\; a^{n-p}\, b^p$$

<u>Examples</u>

1) For all $a, b \in IR$:

$$(a+b)^2 = \sum_{p=0}^{2} C_2^p\; a^{2-p}\, b^p$$
$$= C_2^0\, a^2 b^0 + C_2^1\, a^{2-1} b^1 + C_2^2\, a^{2-2} b^2$$
$$= a^2 + 2ab + b^2$$

2) For all $a \in IR$:

$$(a-2)^5 = \sum_{p=0}^{5} C_5^p a^{5-p} (-2)^p$$
$$= a^5 + 5a^4(-2) + 10a^3(-2)^2 + 10a^2(-2)^3 + 5a(-2)^4 + (-2)^5$$
$$= a^5 - 10a^4 + 40a^3 - 80a^2 + 80a - 32$$

3) For all $a, b \in IR$:

$$(3a-2b)^6 = \sum_{p=0}^{6} C_p^6 (3a)^{6-p} (-2b)^p$$

$$= (3a)^6 + 6(3a)^5(-2b) + 15(3a)^4(-2b)^2 + 20(3a)^3(-2b)^3 + 15(3a)^2(-2b)^4 +$$
$$6(3a)(-2b)^5 + (-2b)^6$$

$$= 729a^6 - 2916a^5b + 4860a^4b^2 - 4320a^3b^3 + 2160a^2b^4 - 576ab^5 + 64b^6$$