L'arbi Ben M'hidi University

Faculty: Exact sciences , natural and life sciences
Department: MI
Academic year: 2023/2024
Module: Algebra 1

Exercise 1 :

Let P, Q, R be three logical propositions. Using a truth table, prove that :

$$(P \vee Q) \wedge R \Longleftrightarrow (P \wedge R) \vee (Q \wedge R)$$

$$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$$

$$\overline{P \wedge Q} \Leftrightarrow \overline{P} \vee \overline{Q}.$$

Exercise 2 :

Let be the following propositions
(1) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : 2x + y > 0.$
(2) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : 2x + y > 0.$
(3) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : 2x + y > 0.$
(4) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R} : 2x + y > 0.$
(5) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : (2x + y > 0 \text{ ou } 2x + y = 0).$
(6) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : (2x + y > 0 \text{ et } 2x + y = 0).$
Are these propositions true or false?

Exercise 3 :

Let the following property P :

$$\forall n \in \mathbb{N}^*, \left(n^2 - 1\right) \text{ is not divisible by } 8 \Rightarrow n \; iseven.$$

i) i) Write the contrapositive of proposition P:
ii) Prove that: any odd integer n is written as $n = 4k + r$ with$r \in \{1, 3\}$ and $k \in \mathbb{N}$.
iii) Then prove the contrapositive.

Exercise 4 :

Show by recurrence that

1) $\forall n \in \mathbb{N}^*, 1^2 + 2^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}.$

2) $\forall n \in \mathbb{N}^*, 1^3 + 2^3 + ... + n^3 = \dfrac{n^2(n+1)^2}{4}.$

Pr. Rezzag.S