Microeconomics

- Utility


## How Do Consumers Make Choices?

- How do you make the best choice in conditions of scarcity?
- In other words, how do you get the "biggest bang for your buck?"
- Consider questions like:
- Why do people purchase more of something when its price falls?
- Why do people buy more goods and services when their budget increases?


## Leaming Objectives

- By the end of this section, you will be able to:
- Expla in margina l utility a nd the signific ance of diminishing marginal utility
- Calculate marginal and total utility
- Propose decisionsthat maximize utility


## Rationality and Self-Interest

- Assumption of Rationality: a lso called the theory of rational beha vior, it is the assumption that people will make choices in their own self-interest.
- The assumption of rationality—also called the theory of rational behavior-is prima rily a simplification that economists make in orderto create a useful model of human decision-making.
- The assumption that individuals a re purely self-interested doesn't imply that individuals are greedy and selfish. People clearly derive satisfaction from helping others, so "self-interest" can also include pursuing things that benefit other people.



## Rationality in Action

## Rationality in Action

Rationa lity suggests that c onsumers will act to maximize self-interest and businesses will act to maximize profits. Both are taking into account the benefits of a choice, given the costs.

## Rationality and Consumers

- When a consumer is thinking about buying a product, what does he or she want? The theory of rational behavior would say that the consumer wants to maximize benefit and minimize cost.
- As the cost of the product increases, it
 becomes less likely that the consumer will decide that the benefits of the purchase outweigh the costs.


## Rationality in Action (cont.)

## Rationality and Students Example

- How do students decide on a major?
- A number of things may factor a student's decision on a major, such as what type of career a student is interested in, the reputation of specific departments at the university a student is attending, and the student's preferences for spec ific fields of study.
- You disc over that Business Analytics majors eam signific a ntly hig her sala ries. This disc overy inc rea ses the benefits in your mind of the Analytic smajor, and you decide to choose that major.


## Rationality and Businesses

- Businesses also have predictable behavior, but rather than seeking to maximize ha ppiness or plea sure, they seek to maximize profits.
- When economists assume that businesses have a goal of maximizing profits, they can make predictionsabout how companies will react to changing business conditions.
- For example: If a company stands to eam more profit by moving some jobsoverseas, then that's the result that economists would predict.


## Consumer Choice and Utility

Table 1. Algerian. Consumption Choices

- Consumerchoice: the combination of goods and services a consumer purchases

| Average Household Income before Taxes | $\mathbf{6 5 0 0 0}$ |
| :--- | :--- |
| Average Annual Expenditures | 60000 |
| Food at home | 12000 |
| Food away from home | 5000 |
| Housing | 15000 |
| Apparel and services | 3000 |
| Transportation | 8000 |
| Healthcare | 4000 |
| Entertainment | 2500 |
| Education | 3500 |
| Personal insurance and pensions | 5000 |
| All else: tobacco, reDZDing, personal care, | 2000 |
| cash contributions, miscellaneous |  |

## Consumer Choice and the Budget Constraint

- Imagine that Ahmed likesto collectTshirts and watch movies
- the quantity of T-shirts is shown on the horizontal axis
- the quantity of movies is on the vertical axis
- The specific choicesalong the budget constraint line show the combinations of affordable T-shirts and movies



## Utility

Table 2. Total Utility

| T-Shirts <br> (Quantity) | Total <br> Utility | M ovies <br> (Quantity) | Total <br> Utility |
| :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 22 | $\mathbf{1}$ | 16 |
| $\mathbf{2}$ | 43 | $\mathbf{2}$ | 31 |
| $\mathbf{3}$ | 63 | $\mathbf{3}$ | 45 |
| $\mathbf{4}$ | 81 | $\mathbf{4}$ | 58 |
| $\mathbf{5}$ | 97 | $\mathbf{5}$ | 70 |
| $\mathbf{6}$ | 111 | $\mathbf{6}$ | 81 |
| $\mathbf{7}$ | 123 | $\mathbf{7}$ | 91 |
| $\mathbf{8}$ | 133 | $\mathbf{8}$ | 100 |

- Uility: the satisfaction or happiness a person gets from consuming a good or service
- Ahmed obta ins utility from consuming T-shirts and consuming movies
- The second column shows the total utility, or total a mount of satisfaction


## Total Utility

- This is a typic al total utility curve showing an inc rease in total utility as consumption of a good increases, though at a decreasing rate
- Total utility follows the expected pattem: it increases as the number of movies that Ahmed watches rises
- Calculate total utility by multiplying the utility of each good by the number of goods, then Dपding that together.
- Three T-shirts are worth 63 utils. Two movies are
 worth 31 utils.
- Total utility of $94(63+31)$.


## Margina I Utility versus Tota I Utility

- A choice at the margin is a decision to do a little more or a little less of something
- Marginal utility is based on the notion that individuals rarely face all-ornothing decisions
- The change in total utility from consuming one more or one less of an item
- The marginal utility of a third slice of pizza is the change in satisfaction one gets when eating the third slice instead of stopping with two
- Marginal thinking: "How much better will I do on an exam if I study for one more hour?"


## Calculating Marginal Utility

- Marginal Utility is equal to the change in total utility divided by the change in qua ntity

$$
M U=\frac{\text { change in total utility }}{\text { change in quantity }}
$$

## Margina I Utility vs. Tota I Utility

- Marginal utility decreases as consumption of a good increases
- This is an example of the law of diminishing marginal utility, which holds that the additional utility dec reases with each unit added
- Diminishing marginal utility is a nother example of the more general law of diminishing retums



## Budget Constraints and Choices

- Budget Constraint refers to all possible combinations of goods that someone can afford, given the prices of goodsand the income (ortime) we have to spend.
- Sunk Costs: costs incurred in the past that can't be recovered.
- Opportunity Cost measures cost by what is given up in exc hange; opportunity cost measures the value of the forgone altemative.

Ahmed's Burgers \& Bus Ticket Budget
Budget: \$10
Burgers: \$2
Bus Tickets: 50 cents


## Budget Constraints and Choices (cont.)

## Types of Budget Constraints

- Limited a mount of money to spend on the things we need a nd want.
- Limited a mount of time.


## Budget Constra ints and Choices (cont. II)

## Budget Constraint Results

- You have to make choices.
- Every choice involvestrade-offs.
- No matter how many goodsa consumerhasto choose from, every choice has an opportunity cost, i.e. the value of the other goods that a ren't chosen.
- The budget constraint framework assumes that sunk costs-costs incurred in the past that can't be recovered-should not affect the current decision.


## Calculating Opportunity Cost Steps

## Steps to Calc ulate Opportunity Cost

- Step 1. Use this equation where $P$ and $Q$ are the price and respective qua ntity of a ny number, n, of items purchased and Budget is the a mount of income one has to spend.
Budget $=P 1 \times Q 1+P 2 \times Q 2+\cdot \cdot+P n \times Q n$
- Step 2. Apply the budget constra int equation to the scenario.
$10=2 \times \mathrm{Q} 1+0.50 \times \mathrm{Q} 2$
- Step 3. Simplify the equation.

We are going solve for $Q_{1}$.

$$
\begin{aligned}
10 & =2 Q_{1}+0.50 Q_{2} \\
10-2 Q_{1} & =0.50 Q_{2} \\
-2 Q_{1} & =-10+0.50 Q_{2} \\
(2)\left(-2 Q_{1}\right) & =(2)-10+(2) 0.50 Q_{2} \\
-4 Q_{1} & =-20+Q_{2} \\
Q_{1} & =5-\frac{1}{4} Q_{2}
\end{aligned}
$$

- Step 4. Use the equation.

$$
\begin{aligned}
& Q_{1}=5-\left(\frac{1}{4}\right) 8 \\
& Q_{1}=5-2 \\
& Q_{1}=3
\end{aligned}
$$

- Step 5. the results.


## Calculating Opportunity Cost - Graph

How many burgers and bus tickets can Ahmed buy?


- Ahmed'sbudged equation: $10=2 \times \mathrm{Q} 1+0.50 \times \mathrm{Q} 2$

| Point | Quantity of <br> Burgers (at \$2) | Quantity of Bus <br> Trckets (at 50 <br> cents) |
| :--- | :--- | :--- |
| A | 5 | 0 |
| B | 4 | 4 |
| C | 3 | 8 |
| D | 2 | 12 |
| E | 1 | 16 |
| F | 0 | 20 |

Burgers: \$2
Bus Tickets: 50 cents


Bus Tickets

## Rules for Maximizing Utility

- Consumerequilibrium: comparing the trade-offs between one affordable combination with all the other affordable combinations
- that is, the combination of goods and servic es that will maximize an individual's total utility
- Ahmed has income of 56 DDD. Movies cost 7 DZ and T-shirts cost 14 DZD. The points on the budget constraint line show the combinations of movies and T-shirts that are affordable



## Applying the Rule

- To maximize total utility, spend each dollar on the item which yields the greatest marginal utility per dollar of expenditure
- Ahmed'sfirst purchase will be a movie. Why?
- Ahmed's choicesare to purchase either a T-shirt or a movie
- The first movie gives Ahmed more marginal utility perdollar than the first T-shirt, and because the movie is within his budget, he will purchase a movie first
- Ahmed will continue to purchase the good which giveshim the highest marginal utility per dollar until he exha usts the budget


## Rules for Maximizing Utility cont.

Table 1. A Step-by-Step Approach to Maximizing Utility

| Try | Which Has | Tota I Utility | Marginal Gain and Loss of Utility, Compared with Previous Choice | Conclusion |
| :---: | :---: | :---: | :---: | :---: |
| Choice 1: P | 4 T-shirts and 0 movies | 81 from 4 T-shirts +0 <br> from 0 movies $=81$ | - | - |
| Choice 2: Q | 3 T-shirts and 2 movies | 63 from 3 T-shirts + 31 from 0 movies $=94$ | Loss of 18 from 1 less T-shirt, but gain of 31 from 2 more movies, fora net utility gain of 13 | $Q$ ispreferred over $P$ |
| Choice 3: R | 2 T-shirtsand 4 movies | 43 from 2 T-shirts +58 from 4 movies $=101$ | Loss of 20 from 1 less $T$-shirt, but gain of 27 from two more moviesfora net utility gain of 7 | $R$ is preferred overQ |
| Choice 4: S | 1 T-shirt and 6 movies | 22 from 1 T-shirt + 81 from 6 movies $=103$ | Loss of 21 from 1 less $T$-shirt, but gain of 23 from two more movies, for a net utility ga in of 2 | S is preferred over R |
| Choice 5: T | 0 T-shirts and 8 movies | 0 from 0 T-shirts +100 from 8 movies $=100$ | Loss of 22 from 1 less $T$-shirt, but gain of 19 from two more movies, for a net utility loss of 3 | S is preferred overT |

## Decision Making by Comparing Margina I Utility

- How Ahmed could use the following thought process (if he thought in utils) to make his decision regarding how many $T$-shirts and moviesto purchase:
- Step 1: From Table 1, Ahmed can see that the marginal utility of the fourth T-shirt is 18. If Ahmed gives up the fourth T-shirt, then he loses 18 utils
- Step 2: Giving up the fourth T-shirt, however, frees up14 DDD (the price of a T-shirt), allowing Ahmed to buy the first two movies (at 7 DZ each)
- Step 3: Ahmed knows that the marginal utility of the first movie is 16 and the marginal utility of the second movie is 15 . Thus, if Ahmed movesfrom point $P$ to point $Q$, he gives up 18 utils (from the T-shirt), but gains 31 utils (from the movies)
- Step 4: Gaining 31 utils a nd losing 18 utils is a net ga in of 13 . This is just a nother way of sa ying that the total utility at Q ( 94 a ccording to the last column in Table 1 ) is 13 more than the total utility at P (81)
- Step 5: So, forAhmed, it makes sense to give up the fourth T-shirt in order to buy two movies


## A Rule for Maximizing Utility

- This process of decision making described previously suggests a rule to follow when maximizing utility
- Since the price of T-shirts is not the same as the price of movies, it's not enough to just compare the marginal utility of T-shirts with the marginal utility of movies
- We need to control forthe prices of each product
- We can do this by computing and comparing marginal utility perdollarof expenditure foreach product
- Marginal utility per dinar is the a mount of additional utility Ahmed receives given the price of the product


## The behaviour of economic actors is often constrained by the economic resources they have at their disposal

- Examples:
- -Individua Is maximising utility will be subject to a budget constraint
- -Firms maximising output will be subject to a cost constraint
- The function we want to maximise is called the objective function
- The restriction is called the constraint

$$
\begin{aligned}
& \operatorname{Max} \mathbf{U}=\mathrm{f}\left(\mathrm{X}_{1} \mathrm{X}_{2}\right)=X_{1}{ }_{1}^{2} X_{2} \\
& {\left[\mathrm{X}_{1}{ }^{*} \mathrm{X}_{2}{ }^{*}\right]}
\end{aligned}
$$

Subject to

$$
\mathrm{g}\left(\mathrm{X}_{1} \mathrm{X}_{2}\right)=\mathrm{P}_{1} \mathrm{X}_{1}+\mathrm{P}_{2} \mathrm{X}_{2}=\mathrm{M}
$$

Two ways to do this:

- By Substitution
- Lagrange Multiplier


## Method 1: By Substitution

Step 1: Use the constraint to express $\mathrm{X}_{2}$ in terms of $X_{1}$ (or vice-versa)

$$
X_{2}=\frac{M}{P_{2}}-\frac{P_{1}}{P_{2}} X_{1}
$$

Step 2: Substitute expression for $\mathbf{X}_{2}$ into the objective function

$$
\begin{aligned}
& U=X_{1}^{2} X_{2}=X_{1}^{2}\left[\frac{M}{P_{2}}-\frac{P_{1}}{P_{2}} X_{1}\right] \\
& \operatorname{Max}_{X_{1}} U=X_{1}^{2} X_{2}=X_{1}^{2} \frac{M}{P_{2}}-X_{1}^{3} \frac{P_{1}}{P_{2}}
\end{aligned}
$$

## Step 3:

$\operatorname{Max}_{X{ }_{1}} U=X_{1}^{2} X_{2}=X_{1}^{2} \frac{M}{P_{2}}-X_{1}^{3} \frac{P_{1}}{P_{2}}$

## F.O. Condition

$$
\begin{aligned}
& d U=f_{1} \cdot d X_{1}=0 \\
& f_{1}=2 X_{1} \frac{M}{P_{2}}-3 X_{1}^{2} \frac{P_{1}}{P_{2}}=0 \\
& 2 \frac{M}{P_{2}}=3 X_{1} \frac{P_{1}}{P_{2}} \\
& X{ }^{*}{ }_{1}=\frac{2}{3} \frac{M}{P_{1}}
\end{aligned}
$$

( $P_{1} X_{1}=2 / 3 M$, expenditure on good 1 is $2 / 3$ of income)

## S. O. Condition

For a Max,

$$
\begin{aligned}
d{ }^{2} U & =f_{11} \cdot d X \quad{ }_{1}^{2}<0 \\
f_{11}= & 2 \frac{M}{P_{2}}-6 X \frac{P_{1}}{P_{2}}
\end{aligned}
$$

$X_{1}$ needs to be large enough to sign N.D.
How Large? First find the $X_{1}$ that sets,

$$
f_{11}=2 \frac{M}{P_{2}}-6 X, \frac{P_{1}}{P_{2}}=0
$$

Answer: $\quad x_{1}=\frac{1}{3} \frac{M}{P_{1}}$
The optimal ${ }_{x}{ }^{*}{ }^{\prime}=\frac{2}{3} \frac{M}{P_{1}} \Longrightarrow \mathbf{f}_{11}<\mathbf{O}$

Step 4: Substitute this value into constraint to find corresponding value of $\mathrm{X}_{2}$ that maximises objective function
Since $P_{1} X_{1}+P_{2} X_{2}=M$

$$
X^{*}{ }_{2}=\frac{M}{P_{2}}-\frac{P_{1}}{P_{2}} X_{1}=\frac{M}{P_{2}}-\frac{P_{1}}{P_{2}}\left[\frac{2}{3} \frac{M}{P_{1}}\right]=\frac{1}{3} \frac{M}{P_{2}}
$$

(note, rearranging, $P_{2} X_{2}=1 / 3 \mathrm{M}$. expenditure on good 2 is $1 / 3$ of $M$ )

## Method 2: By The Lagrange Multiplier

Max the Objective function:
$\operatorname{Max} \mathrm{U}=\mathrm{f}\left(\mathrm{X}_{1} \mathrm{X}_{2}\right)=X_{1^{2}} \mathrm{X}_{2}$
[ $\mathrm{X}_{1}{ }^{*} \mathrm{X}_{2}{ }^{*}$ ]
Subject to the constraint:

$$
g\left(X_{1} X_{2}\right)=P_{1} X_{1}+P_{2} X_{2}-M=0
$$

Step 1: Define the Lagrangean Function L

$$
\begin{aligned}
& \text { (L= objective function }+\lambda \text { constraint }) \\
& \operatorname{Max} \mathrm{L}=\mathrm{f}\left(\mathrm{X}_{1} \mathrm{X}_{2}\right)+\lambda \mathrm{g}\left(\mathrm{X}_{1} \mathrm{X}_{2}\right) \\
& {\left[\mathrm{X}_{1}{ }^{*} \mathrm{X}_{2}^{*} \lambda^{*}\right]} \\
& \operatorname{Max}_{\left[\mathrm{X}_{1}{ }^{*} \mathrm{X}_{2}^{*} \lambda^{*}{ }^{*}\right]}=\mathrm{X}_{1}{ }^{2} \mathrm{X}_{2}+\lambda\left(\mathrm{M}-\mathrm{P}_{1} \mathrm{X}_{1}-\mathrm{P}_{2} \mathrm{X}_{2}\right)
\end{aligned}
$$

$$
\text { OR } \quad L=X_{1}^{2} X_{2}-\lambda\left(\mathrm{P}_{1} \mathrm{X}_{1}+\mathrm{P}_{2} \mathrm{X}_{2}-\mathrm{M}\right)
$$

Step 2: _Find all first order partial derivatives, set $\mathrm{dL}=0$

$$
\begin{array}{lll}
\text { 1. } \mathrm{Lx}_{1}=2 \mathrm{X}_{1} \mathrm{X}_{2}-\lambda \mathrm{P}_{1} & =0 & \text { eq1 } \\
\text { 2. } \mathrm{Lx}_{2}=\mathrm{X}_{1}^{2}-\lambda \mathrm{P}_{2} & =0 & \text { eq2 } \\
\text { 3. } \mathrm{L}_{\lambda}=\mathrm{M}-\mathrm{P}_{1} \mathrm{X}_{1}-\mathrm{P}_{2} \mathrm{X}_{2}=0 & \text { eq3 }
\end{array}
$$

Step 3: Solve the system of equations
Solving equations $1 \& 2$ :
$\lambda=2 \mathrm{X}_{1} \mathrm{X}_{2} / \mathrm{P}_{1}=\mathrm{X}_{1}^{2} / \mathrm{P}_{2}$
so $2 \mathrm{X}_{1} \mathrm{P}_{2} \mathrm{X}_{2}=\mathrm{P}_{1} \mathrm{X}_{1}{ }^{2}$
so $2 \mathrm{P}_{2} \mathrm{X}_{2}=\mathrm{P}_{1} \mathrm{X}_{1}$
expenditure on good 2 is twice that of good 1

```
And substituting into eq 3
\(\mathrm{P}_{1} \mathrm{X}_{1}+\mathrm{P}_{2} \mathrm{X}_{2}-\mathrm{M}=0\)
\(2 \mathrm{P}_{2} \mathrm{X}_{2}+\mathrm{P}_{2} \mathrm{X}_{2}-\mathrm{M}=0\)
\(X_{2}{ }^{*}={ }^{1 / 3}{ }^{\mathrm{M}} / \mathrm{p}_{2}\)
```

and from eq 3:
$\mathrm{X}_{1}={ }^{\mathrm{M}} / \mathrm{P}_{1}-{ }^{\mathrm{P} 2 \mathrm{X} 2} / \mathrm{P} 1$
Substituting in for $X_{2}$ : $\mathrm{X1}^{*}=2 / 3 \mathrm{M} / \mathrm{P} 1$

$$
X_{2}^{*}=\left[\frac{1}{3} \frac{M}{P_{2}}\right] \quad \& X_{1}^{*}=\left[\frac{2}{3} \frac{M}{P_{1}}\right]
$$

(again, note that rearranging reveals that $P_{1} X_{!}=2 / 3$
$M$ and $P_{2} X_{2}=1 / 3 M$.
$2 / 3$ of income spent on good 1 , and $1 / 3$ on good2)

## Step 4: Second Order Condition

$d^{2} L=L_{11} \cdot d X_{1}^{2}+L_{12} \cdot d X_{1} d X_{2}+L_{21} \cdot d X_{2} d X_{1}$
$+L_{22} \mathrm{dX}_{2}^{2}$
s.t. $g_{1} \cdot d X_{1}+g_{2} \cdot d X_{2}=0$
or $\mathrm{dX}_{2}=-\left(g_{1} / g_{2}\right) \cdot d X_{1}$
N. D. for a Max

$$
\begin{aligned}
& d^{2} L=\left[L_{11} \cdot g^{2}-2 L_{12 .} g_{1} \cdot g_{2}+L_{22} g^{2}{ }_{1}\right] d X^{2}{ }_{1} / g_{2}^{2} \\
& \mathbf{d}^{2} \mathbf{L}=\Phi \mathbf{d} X^{2}{ }_{1} / g^{2}{ }_{2} \text {, if } \Phi<O, N . D . \\
& B D=\left|\begin{array}{ccc}
0 & g_{1} & g_{2} \\
g_{1} & L_{11} & L_{12} \\
g_{2} & L_{21} & L_{22}
\end{array}\right|=-\Phi>0 \Rightarrow N^{2} . D \\
& B D=\left|\begin{array}{ccc}
0 & -P_{1} & -P_{2} \\
-P_{1} & 2 X_{2} & 2 X_{1} \\
-P_{2} & 2 X_{1} & 0
\end{array}\right|=-\Phi=2 P_{2} M>0 \Rightarrow N \cdot D \\
& d^{2} L<O, \text { N. D. (Max) }
\end{aligned}
$$

## Example 1

- Question
- A consumers preferencescan be represented by the Utility Function, $\mathbf{U}(\mathbf{x}, \mathbf{y})=\mathbf{x} . \mathbf{y}$.
- How much will the utility maximising consumerdemand of goodsxand y if they have an income of DדD100, the price of good $x$ is 5 D $\triangle$ and the price of good $y$ is 1 Dד?


## Lagrangean Method:

$L=x \cdot y+\lambda[100-5 x-y]$
Eq. 1. $\mathbf{L}_{\mathrm{x}}=\mathrm{y}-5 \lambda=0$
Eq. 2. $\mathrm{L}_{\mathrm{y}}=\mathrm{x}-\lambda=\mathrm{O}$
Eq. 3. $\mathrm{L}_{\lambda}=100-5 x-y=0$
Eq 1\&2: $\quad \lambda={ }^{\mathrm{y}} / 5=\mathrm{x} \Rightarrow \mathrm{y}^{*}=5 \mathrm{x}$ Substitute into eq. 3 :
$100=5 x+y=10 x$
So $\mathrm{x}^{*}=10 \& \mathrm{y}^{*}=5 \mathrm{x}=50 \& \mathrm{U}^{*}=500$ (note that $P_{1} X_{1}=1 / 2 M$ and $P_{2} X_{2}=1 / 2 M$.
1/2 of income spent on good 1, and $1 / 2$ on good2)
Second Order Condition:

$$
B D=\left|\begin{array}{ccc}
0 & -5 & -1 \\
-5 & 0 & 1 \\
-1 & 1 & 0
\end{array}\right|=10>0 \Rightarrow \operatorname{Max}
$$

## Quick Review

- What is utility and its connection to consumer behavior?
- How do you calculate the total utility of a collection of goods and services?
- What is the difference between total and marginal utility?
- Contrast and compute marginal utility and total utility
- Why does maximizing utility require that the last unit of each item purc hased must have the same marginal utility per dollar?
- How do you calculate the utility-maximizing choice?

