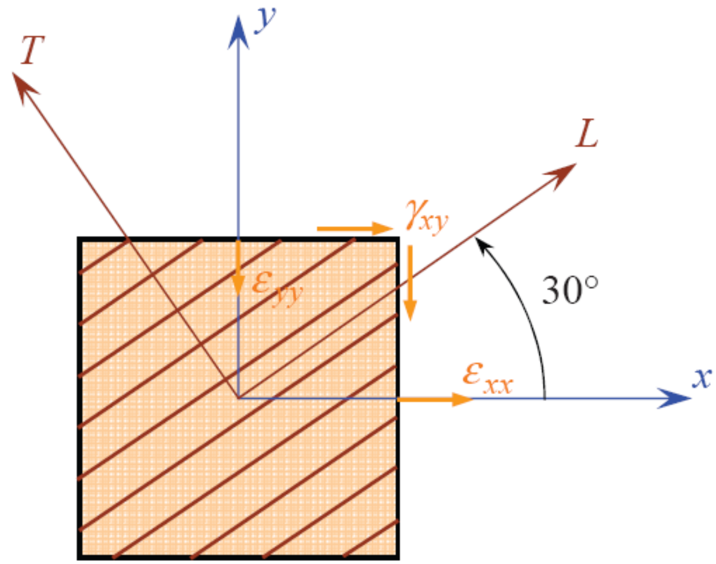
Partie II

Solution Ex 1



1.

Matrice de rigidité dans le plan 1-2

$$\mathbf{Q} = \begin{bmatrix} 41,051 & 3,284 & 0 \\ 3,284 & 10,263 & 0 \\ 0 & 0 & 4,5 \end{bmatrix} \text{GPa}$$

Matrice de rigidité dans le plan *x*-*y*

$$\bar{Q} = \begin{bmatrix} 28.339 & 8.299 & 9.561 \\ 8.299 & 12.945 & 3.770 \\ 9.561 & 3.770 & 9.515 \end{bmatrix} \text{GPa}$$

$$\left[T(\theta)\right]\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

$$\theta = 30° \Rightarrow$$

$$T = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{1}{2}\sqrt{3} \\ \frac{1}{4} & \frac{3}{4} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{2} \end{bmatrix}$$

$$c = \cos(\theta) \text{ et } s = \cos(\theta)$$

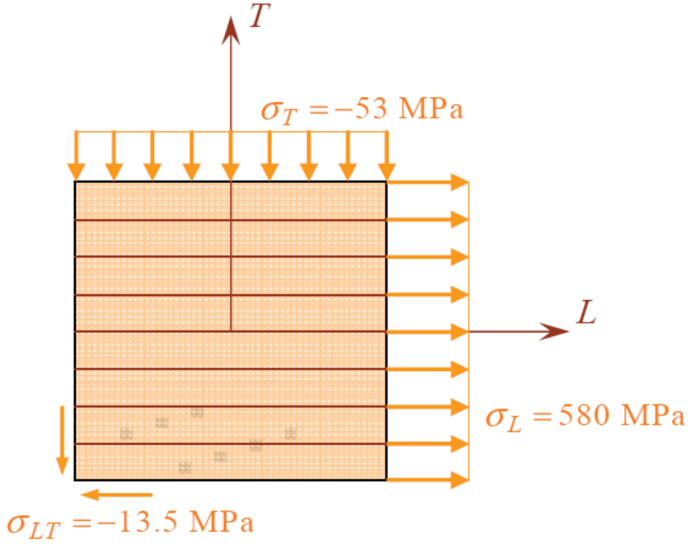
$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}28.339 & 8.299 & 9.561\\ 8.299 & 12.945 & 3.770\\ 9.561 & 3.770 & 9.515\end{bmatrix}\times10^{-9}\begin{bmatrix}10\\ -5\\ 20\end{bmatrix}\times10^{-3} \Rightarrow \begin{matrix}\sigma_x = 433MPa\\ \sigma_y = 94MPa\\ \tau_{xy} = 267MPa\end{matrix}$$

2.

Contraintes dans les axes d'orthropie

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=T\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{Bmatrix}580\\ -53\\ -13.5\end{Bmatrix}MPa$$

Représentation graphique des résultats



3. Comportement thermomécanique

On démontre facilement que la loi de Hook-Duhamel s'écrit dans le système de coordonnées x - y comme suite :

$$\{\varepsilon\}_{(x,y)} = \left[\bar{S}\right]\{\sigma\}_{(x,y)} + \Delta T\,\{\alpha\}_{(x,y)}$$

Avec

$$\{\alpha\}_{(x,y)} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \quad \text{tel que :} \quad \begin{aligned} \alpha_x &= \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta \\ \alpha_y &= \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta \\ \alpha_y &= 2\sin\theta\cos\theta\left(\alpha_2 - \alpha_1\right) \end{aligned}$$

A.N. :

$$\left[\bar{S}\right] = \left[\bar{Q}\right]^{-1} = \begin{bmatrix} 0.0590 & -0.0232 & -0.0501 \\ -0.0232 & 0.0965 & -0.0149 \\ -0.0501 & -0.0149 & 0.1613 \end{bmatrix} GPa^{-1}$$

$$\{\sigma\}_{(x,y)} = \begin{Bmatrix} 580 \\ -53 \\ -13.5 \end{Bmatrix} MPa = \begin{Bmatrix} 580 \\ -53 \\ -13.5 \end{Bmatrix} \times 10^{-3} GPa$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = 10^{-4} \times \begin{Bmatrix} 0.1769 \\ 0.0531 \\ -0.0396 \end{Bmatrix} \frac{1}{°C}$$

On trouve

$$\varepsilon_x = 0.0382$$
$$\varepsilon_y = -0.0178$$
$$\gamma_{xy} = -0.0309$$

Solution Ex 3

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

$$\{\sigma_{(1,2)}\} = [T(\theta)]\{\sigma_{(x,y)}\}$$

$$\{\varepsilon_{(1,2)}\} = [T'(\theta)]\{\varepsilon_{(x,y)}\}$$

Etat de contraintes

$$\begin{Bmatrix} \sigma_x \\ \sigma_y = 0 \\ \tau_{xy} = 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = [T'(\theta)] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & (c^2 - s^2) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\gamma_{12} = -2\cos\theta\sin\theta\,\varepsilon_x + 2\cos\theta\sin\theta\,\varepsilon_y + (\cos^2\theta - \sin^2\theta)\gamma_{xy} \Rightarrow \gamma_{12} = -\varepsilon_x + \varepsilon_y$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2 - s^2) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \tau_{12} = -\cos\theta\sin\theta\,\sigma_x = -\frac{1}{2}\sigma_x$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{-\left(\frac{1}{2}\right)\sigma_x}{-\varepsilon_x + \varepsilon_y} = \frac{\sigma_x}{2(\varepsilon_x + \varepsilon_y)}$$

Solution Ex 5

$$\{\varepsilon\}_{(1,2)} = [S]\{\sigma\}_{(1,2)} = [S]\left[T(\theta)\right]\{\sigma\}_{(x,y)}$$

1. $\{\sigma\}_{(1,2)} = \left[T(\theta)\right]\{\sigma\}_{(x,y)}$

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & \left(c^2 - s^2\right)\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

$$= \begin{bmatrix} 0.75 & 0.25 & 0.866 \\ 0.25 & 0.75 & -0.866 \\ 0.433 & 0.433 & 0.5\end{bmatrix}\begin{Bmatrix}100\\ -50\\ 50\end{Bmatrix} = \begin{Bmatrix}105.8\\ -55.8\\ -39.95\end{Bmatrix}\text{MPa}$$

2. $\{\varepsilon\}_{(1,2)} = [S]\{\sigma\}_{(1,2)}$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1}{70} & -\frac{0.25}{70} & 0 \\ -\frac{0.25}{70} & \frac{1}{70} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{Bmatrix} 105.8 \\ -55.8 \\ -39.95 \end{Bmatrix} \times 10^{-3} = \begin{Bmatrix} 0.00171 \\ -0.00117 \\ -0.808 \end{Bmatrix}$$

Exercice Supplémentaire

Sachant que les propriétés mécaniques du composite sont : E_1= 14GPa, E_2=3.5GPa, G_{12}=4.2GPa, ν_{12}=0.4, ν_{21}=0.1, calculez les déformations selon le système d'axes (x, y).