

Partie I

Solution Ex 1

$$S_{11} = E_1^{-1} = 5,780 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{12} = S_{21} = -\nu_{12} E_1^{-1} = -0,208 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{22} = E_2^{-1} = 30,211 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{13} = S_{31} = -\nu_{13} E_1^{-1} = -1,445 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{33} = E_3^{-1} = 193,424 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{23} = S_{32} = -\nu_{23} E_2^{-1} = -5,166 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{44} = E_4^{-1} = 308,642 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{55} = E_5^{-1} = 120,919 \cdot 10^{-3} \text{GPa}^{-1},$$

$$S_{66} = E_6^{-1} = 106,610 \cdot 10^{-3} \text{GPa}^{-1},$$

Solution Ex 2

Matrice de
passage

$$P = \begin{bmatrix} \cos 30 & \sin 30 & 0 \\ -\sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.87 & 0.5 & 0 \\ -0.5 & 0.87 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrice de
transformation
des contraintes

$$T_{\sigma} = \begin{bmatrix} 0.75 & 0.25 & 0 & 0 & 0 & 0.87 \\ 0.25 & 0.75 & 0 & 0 & 0 & -0.87 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.87 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.87 & 0 \\ -0.435 & 0.435 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$T_{\varepsilon} = (T_{\sigma}^{-1})^T$$

$$C' = T_{\sigma} \cdot C \cdot T_{\sigma}^T$$

$$S' = C'^{-1}$$

$$\mathbf{C} = \begin{bmatrix} 173,415 & 1,425 & 1,334 & 0 & 0 & 0 \\ 1,425 & 33,271 & 0,899 & 0 & 0 & 0 \\ 1,334 & 0,899 & 5,205 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3,24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8,27 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9,38 \end{bmatrix} \text{GPa,}$$

$$\mathbf{S} = \begin{bmatrix} 5,780 & -0,208 & -1,445 & 0 & 0 & 0 \\ -0,208 & 30,211 & -5,166 & 0 & 0 & 0 \\ -1,445 & -5,166 & 193,424 & 0 & 0 & 0 \\ 0 & 0 & 0 & 308,642 & 0 & 0 \\ 0 & 0 & 0 & 0 & 120,919 & 0 \\ 0 & 0 & 0 & 0 & 0 & 106,610 \end{bmatrix} 10^{-3} \text{GPa}^{-1}$$

$$\{\boldsymbol{\varepsilon}\} = \mathbf{T}_{\boldsymbol{\varepsilon}}^{-1} \{\boldsymbol{\varepsilon}'\}$$

$$\{\boldsymbol{\sigma}\} = \mathbf{C} \{\boldsymbol{\varepsilon}\}$$

$$\{\boldsymbol{\sigma}'\} = \mathbf{C}' \{\boldsymbol{\varepsilon}'\}$$

Solution Ex 3

**Matériau isotrope
transverse avec :
Plan d'isotropie
1-2**

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12}) \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{pmatrix}$$

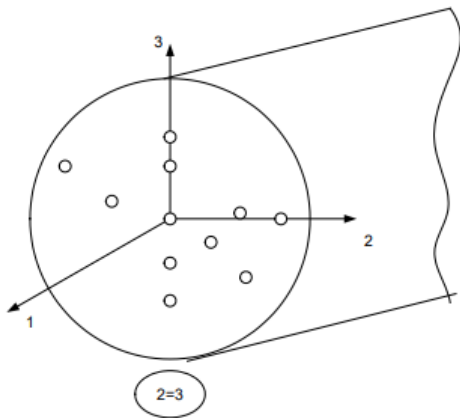
$$S_{11} = S_{22} \Rightarrow \frac{1}{E_1} = \frac{1}{E_2} \text{ donc } E_1 = E_2$$

$$S_{13} = S_{23} \Rightarrow \frac{\nu_{31}}{E_3} = \frac{\nu_{32}}{E_3} \text{ donc } \nu_{31} = \nu_{32}$$

$$S_{31} = S_{32} \Rightarrow \frac{\nu_{13}}{E_1} = \frac{\nu_{23}}{E_2} \text{ donc } \nu_{13} = \nu_{23}$$

$$S_{44} = S_{55} \Rightarrow G_{23} = G_{13}$$

$$S_{66} = 2(S_{11} - S_{12}) \Rightarrow \frac{1}{G_{12}} = 2 \left(\frac{1}{E_1} + \frac{\nu_{12}}{E_1} \right) \Rightarrow G_{12} = \frac{E_1}{2(1 + \nu_{12})}$$



$$G_{13} = G_{12}$$

$$E_2 = E_3$$

$$\nu_{21} = \nu_{31}$$

$$\nu_{23} = \nu_{32}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{21}/E_2 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{23}/E_2 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & -\nu_{23}/E_2 & 1/E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

Solution Ex 6

$$\text{Tableau 2.3} \Rightarrow \left\{ \begin{array}{l} \text{AS/3501} \\ \text{Graphite} \\ \text{Expoxy} \end{array} \right. \left\{ \begin{array}{l} E_1 = 138 \text{ GPa} \\ E_2 = 9 \text{ GPa} \\ G_{12} = 6.9 \text{ GPa} \\ \nu_{12} = 0.3 \end{array} \right. \quad \begin{array}{l} \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = \nu_{12} \frac{E_2}{E_1} \\ = 0.3 \times \frac{9}{138} = 0.0196 \end{array}$$

Matrice de rigidité $[Q]$:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = \frac{138}{1 - 0.3 \times 0.0196} = 138.8 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{0.3 \times 9}{1 - 0.3 \times 0.0196} = 2.716 \text{ GPa} = Q_{21}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} = \frac{9}{1 - 0.3 \times 0.0196} = 9.05 \text{ GPa}$$

$$Q_{66} = G_{12} = 6.9 \text{ GPa}$$

$$S_{11} = \frac{1}{E_1} = \frac{1}{138} = 0.00725 \text{ (GPa)}^{-1}$$

Matrice de souplesse $[S]$:

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.3}{138} = -0.00217 \text{ (GPa)}^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{9} = 0.111 \text{ (GPa)}^{-1}$$

TABLEAU 2.3 Constantes élastiques fondamentales des composites usuels [Gibson]

Composites	E_1		E_2		G_{12}		ν_{12}	ν_f
	Msi	(GPa)	Msi	(GPa)	Msi	(GPa)		
T300/934 Graphite/epoxy	19.0	(131)	1.5	(10.3)	1.0	(6.9)	0.22	0.65
AS/3501 Graphite/epoxy	20.0	(138)	1.3	(9.0)	1.0	(6.9)	0.3	0.65
p-100/ERL 1962 pitch graphite/expoxy	68.0	(468.9)	0.9	(6.2)	0.81	(5.58)	0.31	0.62
Kevlar [®] 49/934 aramid/epoxy	11.0	(75.8)	0.8	(5.5)	0.33	(2.3)	0.34	0.65
Scotchply [®] 1002 E-glass/epoxy	5.6	(38.6)	1.2	(8.27)	0.6	(4.14)	0.26	0.45
Boron/5505 Boron/epoxy	29.6	(204)	2.68	(18.5)	0.81	(5.59)	0.23	0.5
Spectra [®] 900/826 Polyethylene/epoxy	4.45	(30.7)	0.51	(3.52)	0.21	(1.45)	0.32	0.65
E-glass/470-36 E-glass/vinylester	3.54	(24.4)	1.0	(6.87)	0.42	(2.89)	0.32	0.30

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