
2nd Tutorial Series ”Improper Integrals” 2nd Year Engineering (S3)

Exercise 1:

Study the convergence of the following improper integrals:

1. $I_1 = \int_3^4 \frac{1}{(x-3)^3} dx,$
2. $I_2 = \int_0^5 \frac{x+1}{x(x-5)} dx,$
3. $I_3 = \int_0^{+\infty} \frac{1}{1+x^2} dx,$
4. $I_4 = \int_{-1}^2 \frac{1}{x^3} dx,$
5. $I_5 = \int_{-3}^2 \frac{1}{x^2} dx,$
6. $I_6 = \int_2^{+\infty} \frac{1}{x(\ln x)^2} dx.$

Exercise 2:

Using the ”Comparison Test,” study the convergence of the following improper integrals:

1. $I_1 = \int_0^{+\infty} \frac{1}{2+|\cos x|} dx,$
2. $I_2 = \int_0^{+\infty} \frac{1-\cos x}{x^2} dx,$
3. $I_3 = \int_1^{+\infty} \frac{e^{\sin x}}{\sqrt{x}} dx,$
4. $I_4 = \int_1^5 \frac{1}{\sqrt{(5-x)(x+1)}} dx,$
5. $I_5 = \int_0^1 \frac{1}{x \sin x} dx,$
6. $I_6 = \int_0^{+\infty} \frac{\sin^2 x}{1+x^2} dx.$

Exercise 3:

Show that the following integrals are absolutely convergent:

1. $I_1 = \int_0^1 \frac{\cos x}{\sqrt{x}} dx,$
2. $I_2 = \int_1^{+\infty} \frac{\cos x}{x^3} dx,$
3. $I_3 = \int_1^{+\infty} \frac{e^{\sin x}}{\sqrt{x}} dx,$