Ministry of Higher Education and Scientific Research
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Computer Structure 1

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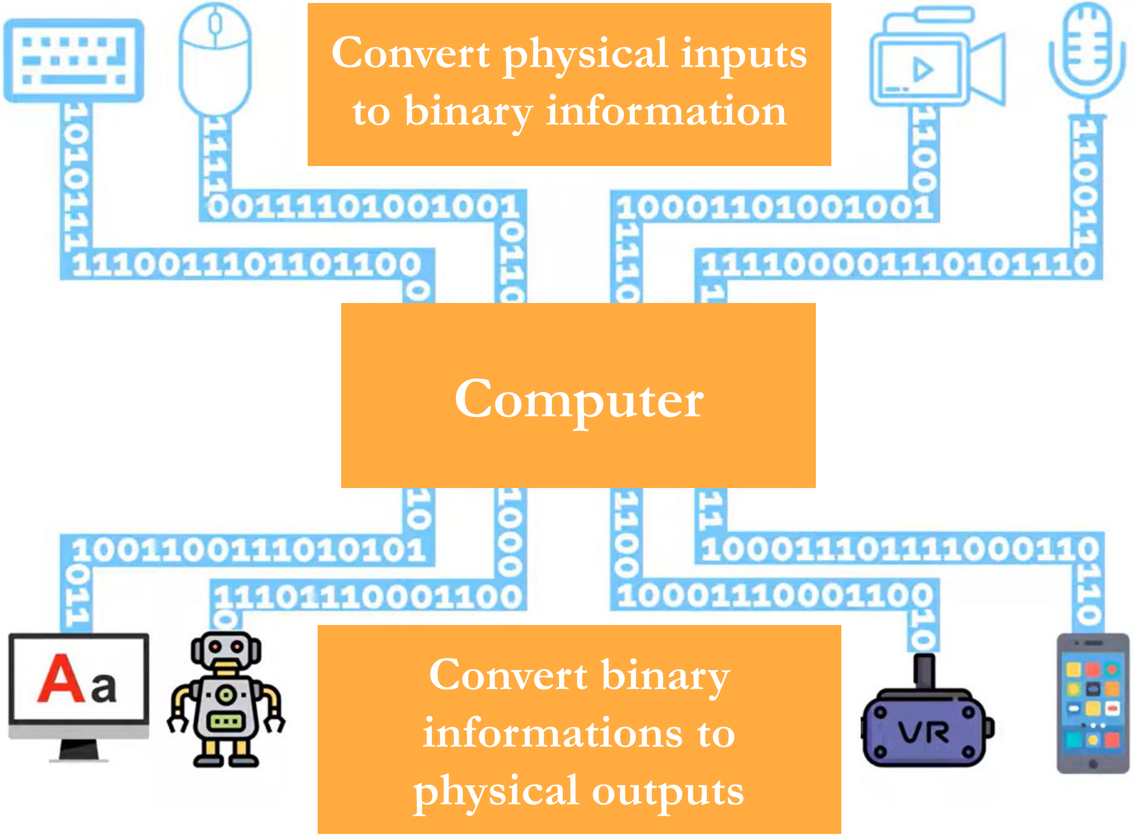
2023-2024

Chapter 1: Numeral systems

Introduction



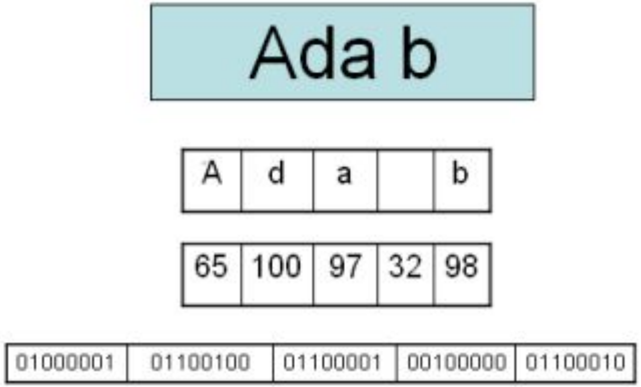
Convert physical inputs to binary information

Computer

Convert binary informations to physical outputs

Introduction

Coding information => creating a **correspondence** between the (normal) **external** representation and its **internal** representation in the computer



Ada b

A	d	a		b
65	100	97	32	98
01000001	01100100	01100001	00100000	01100010

Example of encoding of the character string ”Ada b”.

Introduction

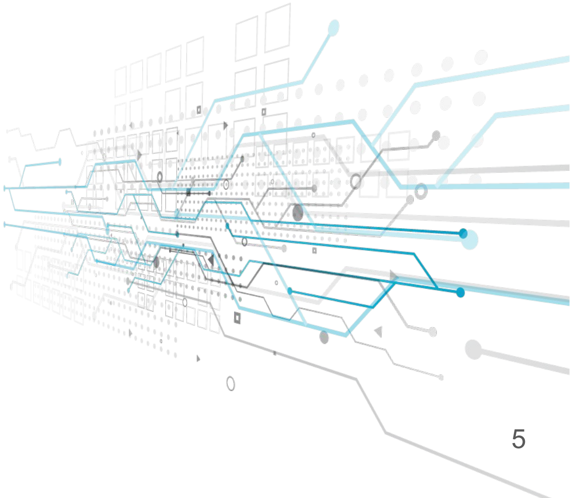
Base 2

Base 8

Base 10

Base 16

The binary system which uses two different numbers: 0, 1



Introduction

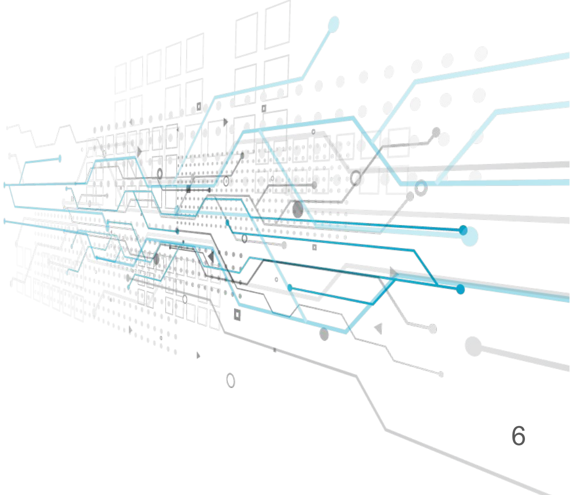
Base 2

Base 8

Base 10

Base 16

The octal system which uses eight different digits: 0 to 7



Introduction

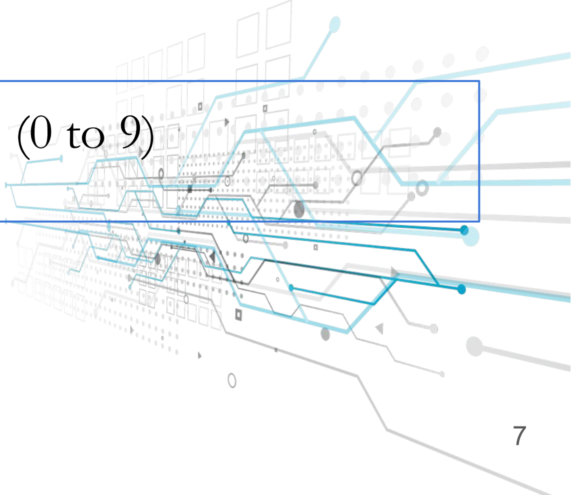
Base 2

Base 8

Base 10

Base 16

The decimal system using ten different digits (0 to 9)



Introduction

Base 2

Base 8

Base 10

Base 16

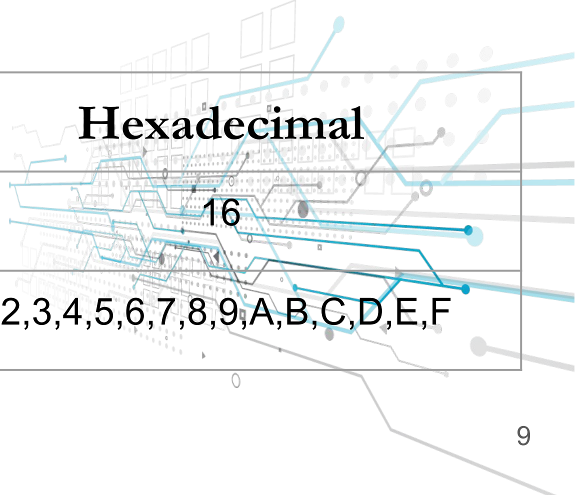
The hexadecimal system which uses sixteen different numbers: from 0 to 9, in addition to five characters A, B, C, D, E, F

Numeral system is characterized by:

- A base B>1.

- Different coefficients or symbols $\mathbf{a}_i$ such as: $0 \leq \mathbf{a}_i < B$.

	Binary	**Octal**	**Decimal**	**Hexadecimal**
Base	2	8	10	16
coefficients	0,1	0,1,2,3,4,5,6,7	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F



B-based system

Base 10

B_{10}	2	0	2	3
Rank of the number	3	2	1	0
Power of base	10^3	10^2	10^1	10^0
	1000	100	10	1

B-based system

Base 10

N_{10}	2	0	2	3
Rank of the number	3	2	1	0
Power of base	10^3	10^2	10^1	10^0
	1000	100	10	1

$$2023 = 3\times10^0 + 2\times10^1 + 0\times10^2 + 2\times10^3$$

B-based system

Base b

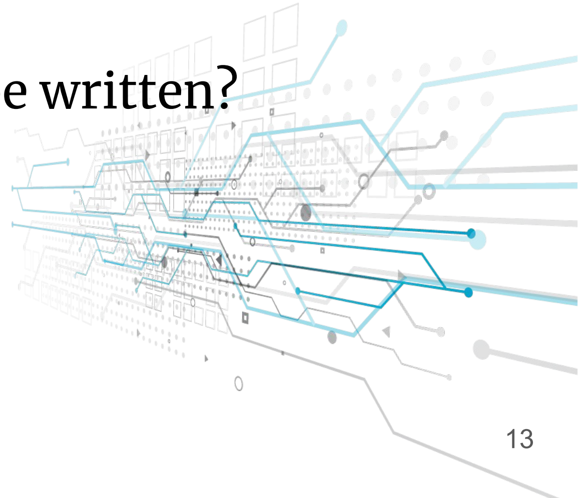
Generalization					
N_b	…	a_3	a_2	a_1	a_0
Rank of the number	…	3	2	1	0
Power of base	…	b^3	b^2	b^1	b^0

$$N_b = a_0 \times b^0 + a_1 \times b^1 + a_2 \times b^2 + a_3 \times b^3 + \ldots$$

Exercise:

Here are the given numbers: 1010, 1020, 108141, 2A0GF00, 01AFB, CEE, BAC.

- Among these numbers, which ones can be the presentation of a number in base 2, 8, 10 or 16?
- Give the smallest base in which each number can be written?



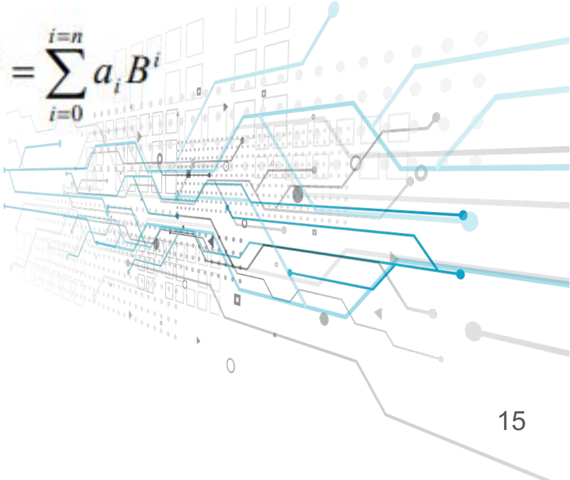
Solution

	Base 2	Base 8	Base 10	Base 16	Smallest Base
1010	**Yes**	**Yes**	**Yes**	**Yes**	**2**
1020	**No**	**Yes**	**Yes**	**Yes**	**3**
108141	**No**	**No**	**Yes**	**Yes**	**9**
2A0GF00	**No**	**No**	**No**	**No**	**17**
01AFB	**No**	**No**	**No**	**Yes**	**16**
CEE	**No**	**No**	**No**	**Yes**	**15**
BAC	**No**	**No**	**No**	**Yes**	**13**

Polynomial Form:

$$N = (\underset{\substack{\downarrow \\ \text{strong weight}}}{a_n} a_{n-1} a_{n-2} \ldots\ldots a_1 \underset{\substack{\downarrow \\ \text{low weight}}}{a_0})_B$$

$$N = a_n B^n + a_{n-1} B^{n-1} + \ldots\ldots + a_1 B^1 + a_0 B^0 = \sum_{i=0}^{i=n} a_i B^i$$

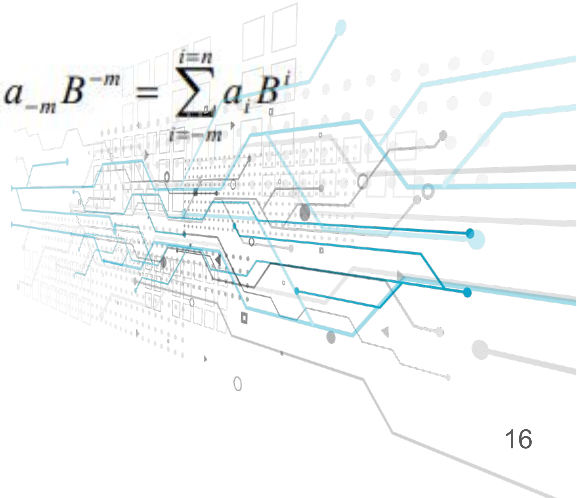


B-based system

Polynomial Form:

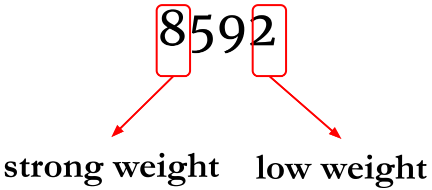
$$N = (\underbrace{a_n a_{n-1} a_{n-2} \dots\dots\dots a_1 a_0}_{\textbf{Integer Part}}, \underbrace{a_{-1} a_{-2} \dots\dots\dots a_{-m}}_{\textbf{Decimal part}})_B$$

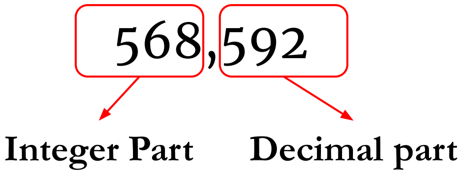
$$N = a_n B^n + a_{n-1} B^{n-1} + \dots\dots + a_1 B^1 + a_0 B^0 + a_{-1} B^{-1} + a_{-2} B^{-2} + \dots\dots + a_{-m} B^{-m} = \sum_{i=-m}^{i=n} a_i B^i$$



B-based system

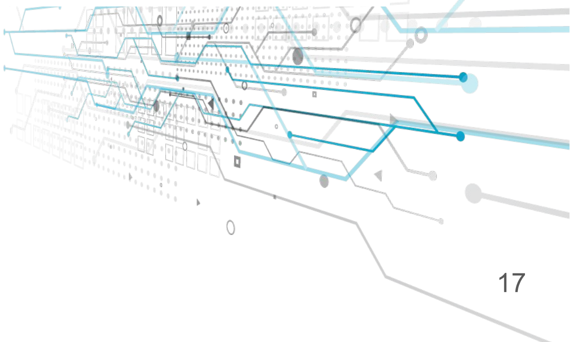
Example:





$8592 = 8x10^3 + 5x10^2 + 9x10^1 + 2x10^0.$

$568.592 = 5x10^2 + 6x10^1 + 8x10^0 + 5x10^{-1} + 9x10^{-2}$
$+2x10^{-3}.$



Converting from base B to decimal:

Method 1: Polynomial expansion

- Express the number in polynomial form using base b,
- then sum the various terms of the polynomial representation of the number.

Example:

$$(12)_3 = 1 \times 3^1 + 2 \times 3^0 = 3 + 2 = (5)_{10}$$

$$(101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 32 + 8 + 4 + 1 = (45)_{10}$$

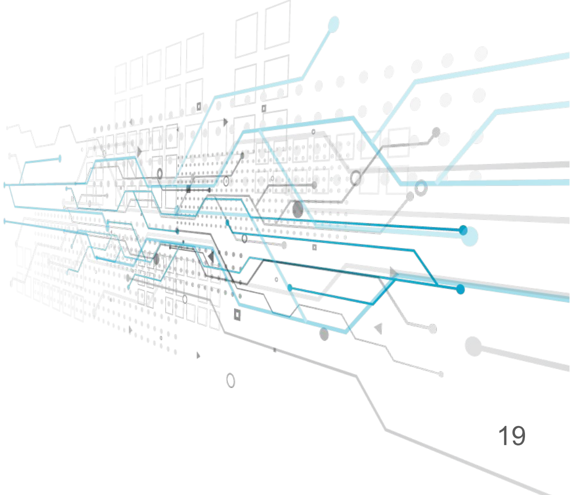
Converting from base B to decimal:

Example:

$(111{,}01011)_2 =$

$(1254{,}1)_8 =$

$(A5F{,}6)_{16} =$



Converting from base B to decimal:

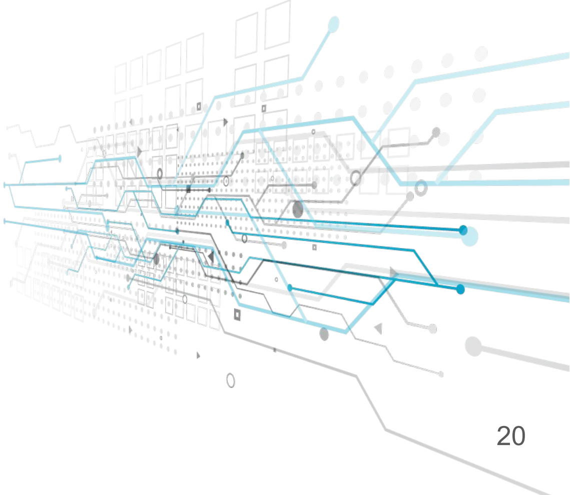
Example:

$$(111{,}01011)_2 = 1\times2^{2} + 1\times2^{1} + 1\times2^{0} + 0\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4} + 1\times2^{-5}$$

$$= 4+2+1+0+\tfrac{1}{4}+0+1/16+1/32 = (7,\ 34375)_{10}$$

$(1254{,}1)_8 =$

$(A5F{,}6)_{16} =$



Converting from base B to decimal:

Example:

$$(111{,}01011)_2 = 1\times2^{2} + 1\times2^{1} + 1\times2^{0} + 0\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4} + 1\times2^{-5}$$

$$= 4+2+1+0+1/4+0+1/16+1/32 = (7,\ 34375)_{10}$$

$$(1254{,}1)_8 = 1\times8^{3} + 2\times8^{2} + 5\times8^{1} + 4\times8^{0} + 1\times8^{-1} = 512+128+40+4+1/8 = (684{,}125)_{10}$$

$$(A5F{,}6)_8 =$$

Converting from base B to decimal:

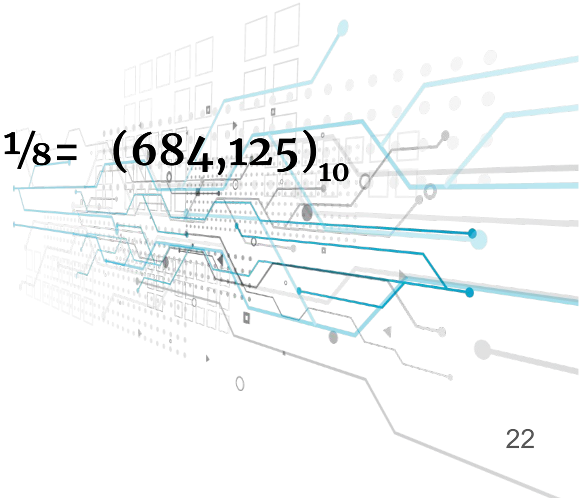
Example:

$$(111{,}01011)_2 = 1\times2^2 + 1\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4} + 1\times2^{-5}$$

$$= 4+2+1+0+\tfrac{1}{4}+0+1/16+1/32 = (7{,}\ 34375)_{10}$$

$$(1254{,}1)_8 = 1\times8^3 + 2\times8^2 + 5\times8^1 + 4\times8^0 + 1\times8^{-1} = 512+128+40+4+\tfrac{1}{8} = (684{,}125)_{10}$$

$$(A5F{,}6)_{16} = 10\times16^2 + 5\times16^1 + 15\times16^0 + 6\times16^{-1} = (2655{,}375)_{10}$$

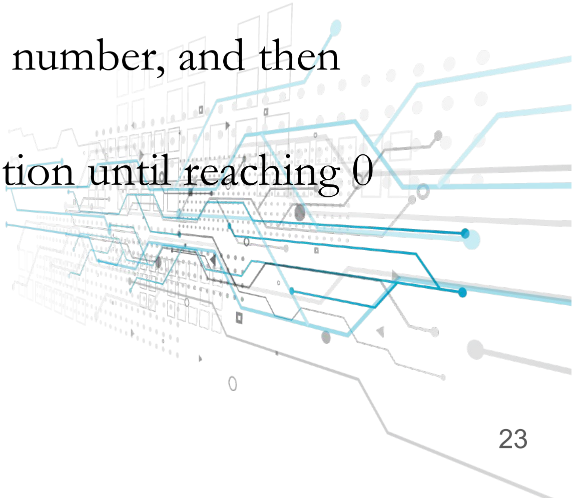


Converting from decimal to base B:

Converting Integer Part

Method 1: Successive Subtractions

- Begin by determining the nearest power of B to the decimal number, and then subtract this power from the number
- Then, repeat the same process with the result of the subtraction until reaching 0



Converting from decimal to base B:

Converting Integer Part

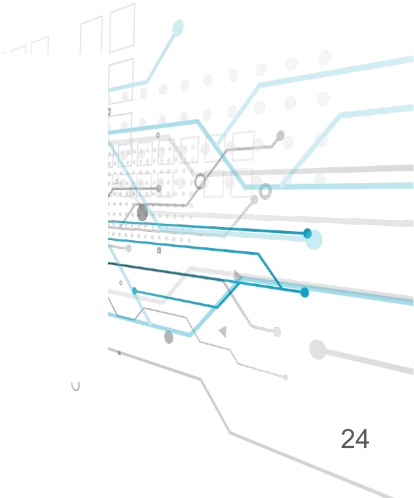
Method 1: Successive Subtractions

Example 1:

$(5)_{10} = (?)_2$

$$5 - 4 = 1 - 1 = 0; donc\ 5 = 4 + 1$$

$$5 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (101)_2$$



Converting from decimal to base B:

Converting Integer Part

Method 1: Successive Subtractions

Example 2:

$(45)_{10} = (?)_2$

$$45 - 32 = 13 - 8 = 5 - 4 = 1 - 1 = 0; donc\ 45 = 32 + 8 + 4 + 1$$

$$45 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (101101)_2$$

Converting from decimal to base B:

Converting Integer Part

Method 1: Successive Subtractions

Example 3:

$$(555)_{10} = (?)_2$$

$$555 - 512 = 43 - 32 = 11 - 8 = 3 - 2 = 1 - 1 = 0; \textit{donc}\ 555 = 512 + 32 + 8 + 2 + 1$$

$$555 = 1 \times 2^9 + 0 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = (1000101011)_2$$

Converting from decimal to base B:

Converting Integer Part

Method 1: Successive Subtractions

Example 3:

$$(555)_{10} = (?)_9 = (?)_8 = (?)_{16}$$

It is not easy to find the first power of 8, 16 or 9 close to 555 !!!

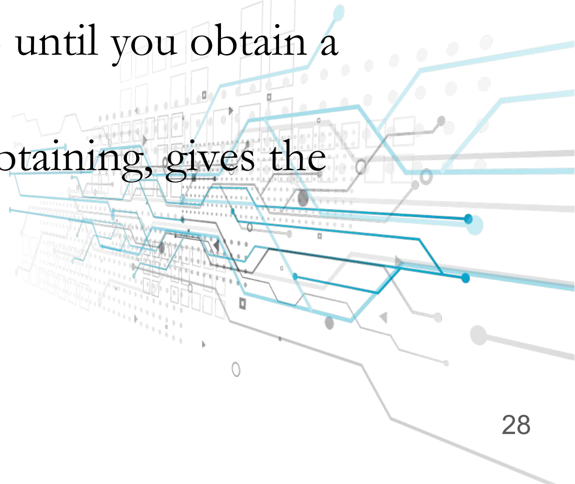
This is the main disadvantage of this method.

Converting from decimal to base B:

Converting Integer Part

Method 2: Successive division

- Divide the whole number and each successive quotient by B until you obtain a null quotient.
- The sequence of remainders, in the reverse order of their obtaining, gives the representation of the decimal number in the base system B.

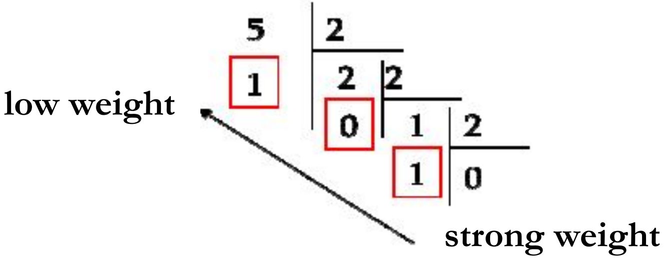


Converting from decimal to base B:

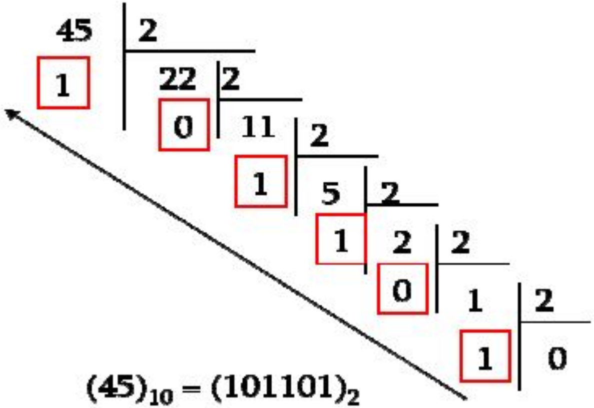
Converting Integer Part

Method 2: Successive division

Example 1:



$(5)_{10} = (101)_2$



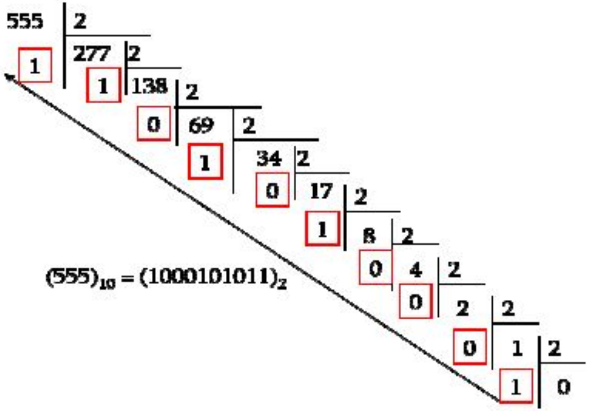
$(45)_{10} = (101101)_2$

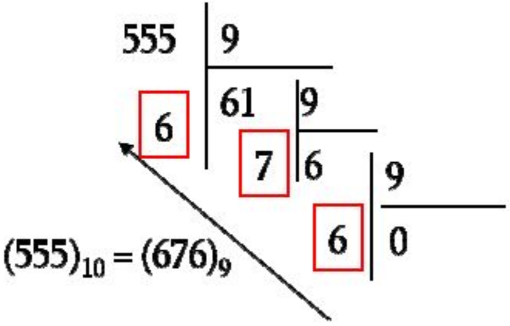
Converting from decimal to base B:

Converting Integer Part

Method 2: Successive division

Example 2:



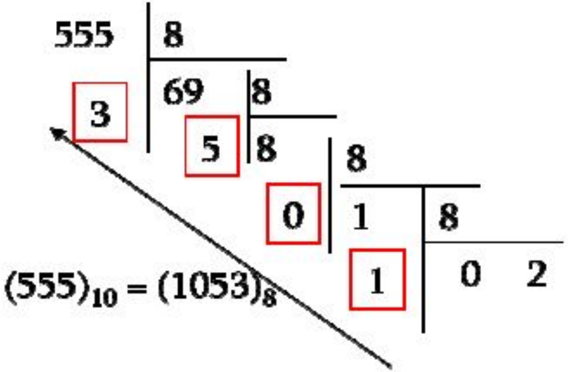


Converting from decimal to base B:

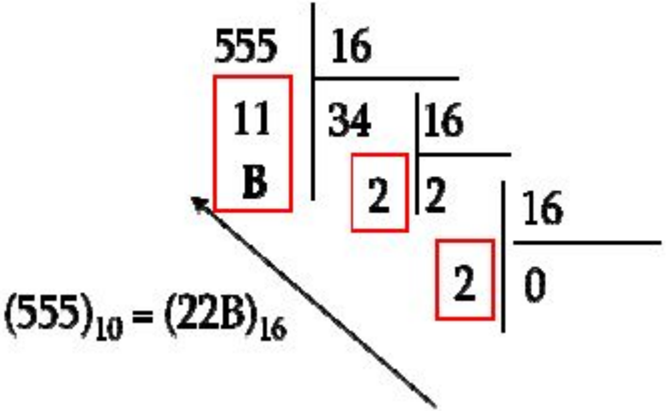
Converting Integer Part

Method 2: Successive division

Example 3:



$(555)_{10} = (1053)_8$



$(555)_{10} = (22B)_{16}$

Converting from decimal to base B:

Converting Decimal Part

Method 2: Successive multiplications

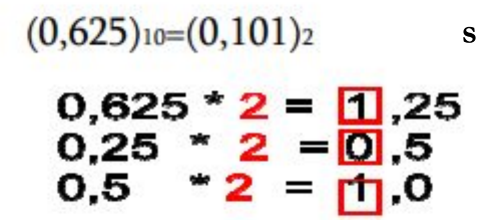
- Multiply the decimal part and the decimal parts of successive products by the base B until you obtain either a null decimal part or a repetition of one of the decimal parts.
- The finite, or infinitely repeated, sequence of the whole parts of the products obtained constitutes the representation of the decimal part in base b.

Converting from decimal to base B:

Converting Decimal Part

Method 2: Successive multiplications

Example1:



$(0,625)_{10}=(0,101)_2$

$0,625 * 2 = \boxed{1},25$

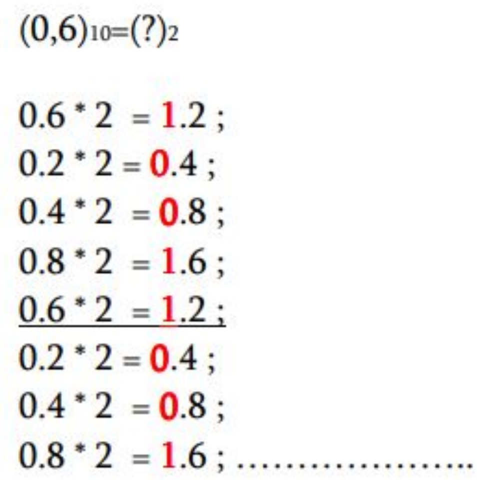
$0,25 * 2 = \boxed{0},5$

$0,5 * 2 = \boxed{1},0$

strong weight

↓

low weight



$(0,6)_{10}=(?)_2$

$0.6 * 2 = 1.2 ;$

$0.2 * 2 = 0.4 ;$

$0.4 * 2 = 0.8 ;$

$0.8 * 2 = 1.6 ;$

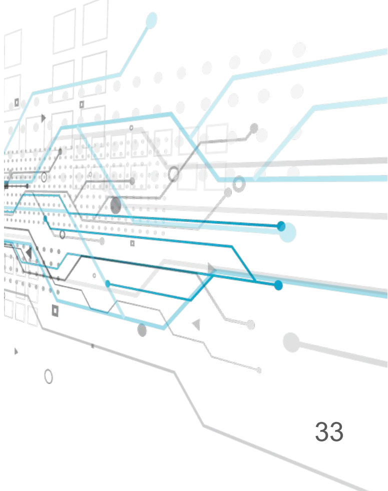
$0.6 * 2 = 1.2 ;$

$0.2 * 2 = 0.4 ;$

$0.4 * 2 = 0.8 ;$

$0.8 * 2 = 1.6 ;$ ………………….

$(0.6)_{10}= (0.10011001)_2$



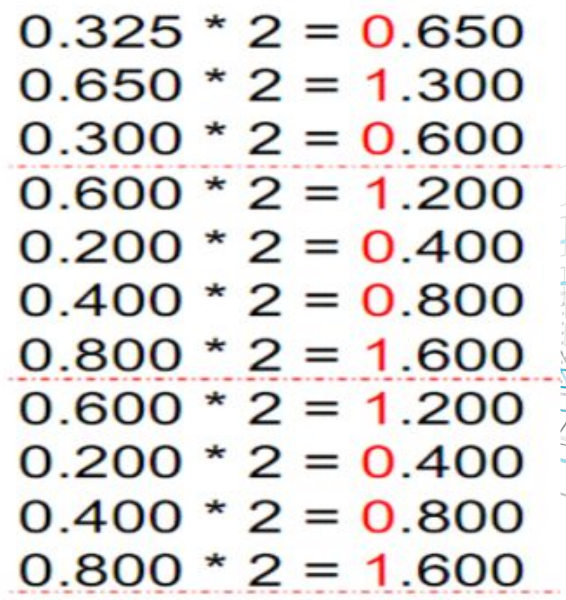
Converting from decimal to base B:

Converting Decimal Part

Method 2: Successive multiplications

Example2:

$(0.325)_{10}=(?)_2$

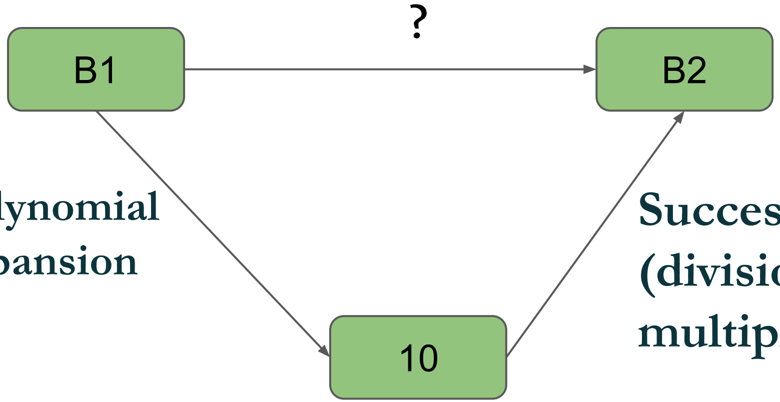


0.325 * 2 = 0.650
0.650 * 2 = 1.300
0.300 * 2 = 0.600
0.600 * 2 = 1.200
0.200 * 2 = 0.400
0.400 * 2 = 0.800
0.800 * 2 = 1.600
0.600 * 2 = 1.200
0.200 * 2 = 0.400
0.400 * 2 = 0.800
0.800 * 2 = 1.600

$(0.325)_{10}=(0101001)_2$

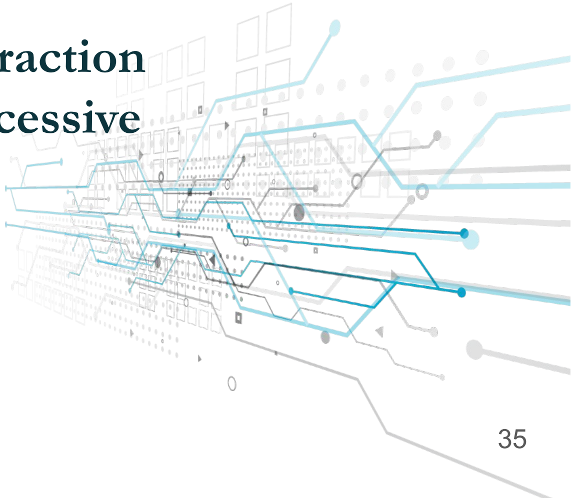
$(0.325)_{10}=(01010011001)_2$.

Converting from base B1 to base B2:



Polynomial expansion

Successive subtraction (division)+ Successive multiplication

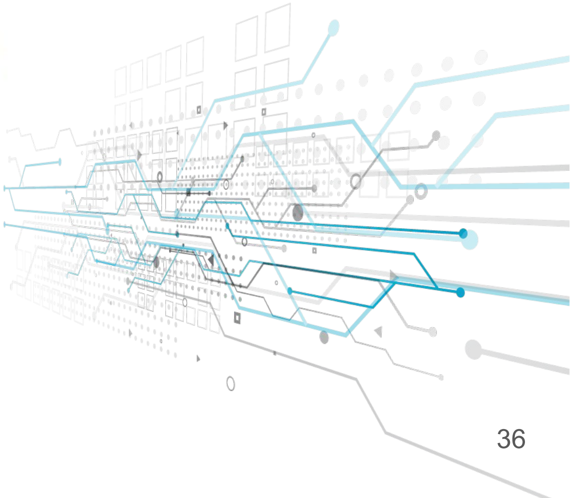


Converting from base B1 to base B2:

Example1:

$$(32{,}4)_5 = (?)_2$$

$$(32{,}4)_5 = 3 \times 5^1 + 2 \times 5^0 + 4 \times 5^{-1} = (17{,}8)_{10} = (10001{,}1100 \ldots \ldots)_2$$



Converting from base B1 to base B2:

Case of bases B and B^k (binary=>octal=>hexadecimal)

binary=>octal (do bursts on 3 bits)

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

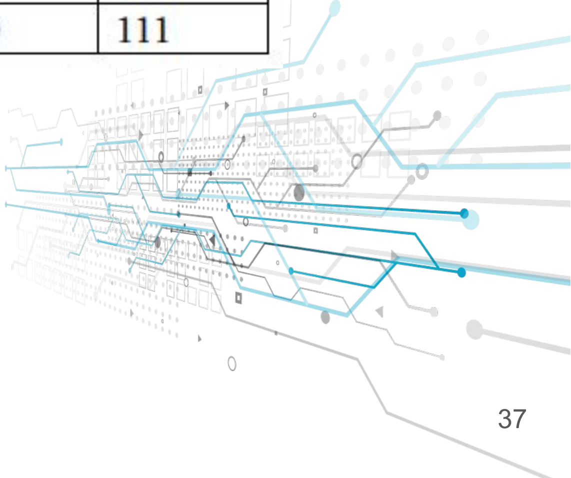
Example:

$(37)_8 = (\underline{011}\ \underline{111})_2 = (11\ 111)_2$

$(1254\ ,1)_8 = (\underline{001}\ \underline{010}\ \underline{101}\ \underline{100},\ \underline{001})_2 = (1\ 010\ 101\ 100,\ 001)_2$

$(1000101011)_2 = (\underline{001}\ \underline{000}\ \underline{101}\ \underline{011})_2 = (1053)_8$

$(1111000{,}01)_2 = (\underline{001}\ \underline{111}\ \underline{000},\ \underline{010})_2 = (170{,}2)_8$



Converting from base B1 to base B2:

Case of bases B and B^k (binary=>octal=>hexadecimal)

binary=>hexadecimal (do bursts on 4 bits)

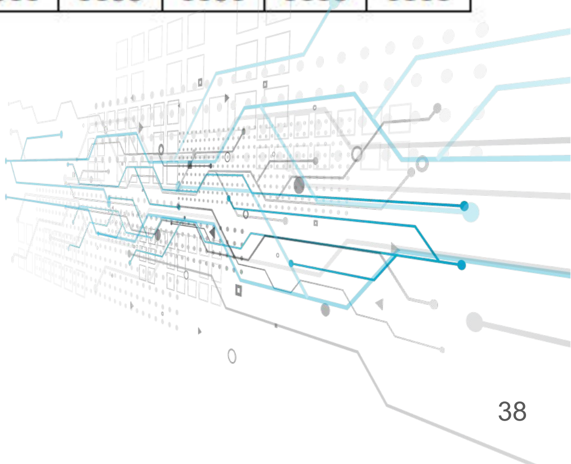
Hexa	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

Example:

$$(A5F,6)_{16} = (\underline{1010}\ \underline{0101}\ \underline{1111},\ \underline{0110})_2$$

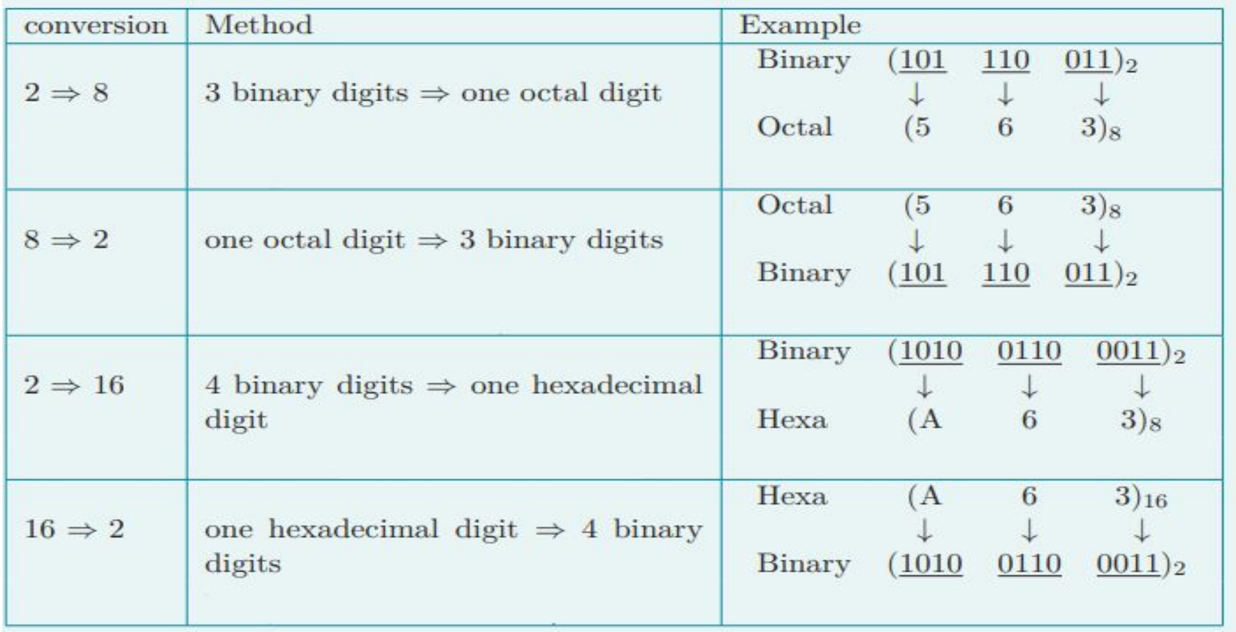
$$(1000101011)_2 = (\underline{0010}\ \underline{0010}\ \underline{1011})_2 = (22B)_{16}$$

$$(1111000,01)_2 = (\underline{0111}\ \underline{1000},\ \underline{0100})_2 = (78,4)_{16}$$

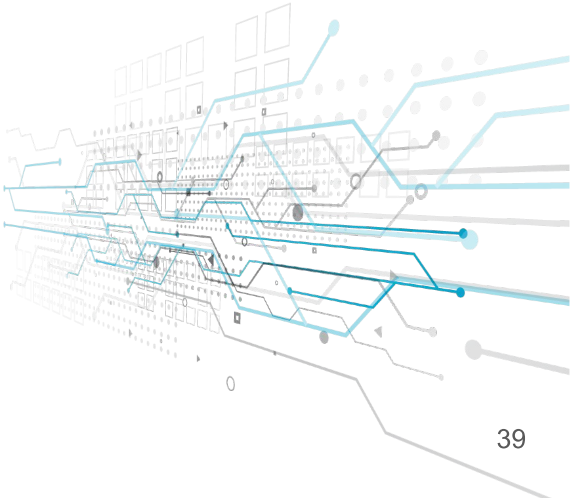


Converting from base B1 to base B2:

Case of bases B and B^k (binary=>octal=>hexadecimal)



conversion	Method	Example
2 ⇒ 8	3 binary digits ⇒ one octal digit	Binary $(\underline{101}\ \ \underline{110}\ \ \underline{011})_2$ $\downarrow\ \ \downarrow\ \ \downarrow$ Octal $(5\ \ 6\ \ 3)_8$
8 ⇒ 2	one octal digit ⇒ 3 binary digits	Octal $(5\ \ 6\ \ 3)_8$ $\downarrow\ \ \downarrow\ \ \downarrow$ Binary $(\underline{101}\ \ \underline{110}\ \ \underline{011})_2$
2 ⇒ 16	4 binary digits ⇒ one hexadecimal digit	Binary $(\underline{1010}\ \ \underline{0110}\ \ \underline{0011})_2$ $\downarrow\ \ \downarrow\ \ \downarrow$ Hexa $(A\ \ 6\ \ 3)_8$
16 ⇒ 2	one hexadecimal digit ⇒ 4 binary digits	Hexa $(A\ \ 6\ \ 3)_{16}$ $\downarrow\ \ \downarrow\ \ \downarrow$ Binary $(\underline{1010}\ \ \underline{0110}\ \ \underline{0011})_2$



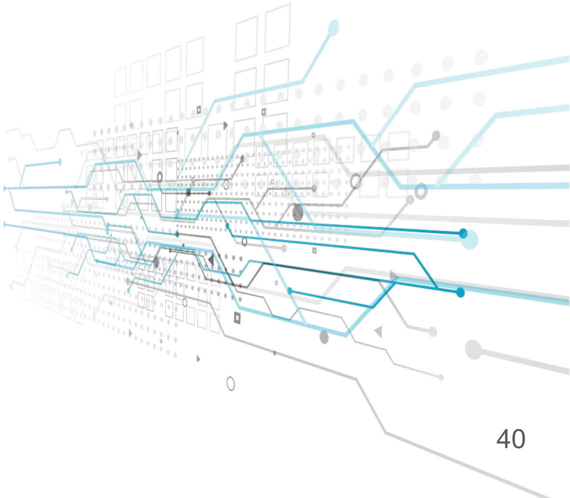
Converting a large decimal number to binary

To convert a large decimal number to binary, it's recommended to initially convert it to **octal** or **hexadecimal**, and then proceed to binary conversion => to reduce the number of operations (division or subtraction).

Example:

$(25693)_{10} = (?????????????)_2$

- **Method 1:** Successive divisions by 2
- **Method 2:** Successive subtractions
- **Method 3:** Convert to base 8 first
- **Method 4:** Convert to base 16 first



Converting a large decimal number to binary

Example:

$(25693)_{10} = (110010001011101)_2$

- **Method 1:** Successive divisions by 2 => **15 division operations**
- **Method 2:** Successive subtractions => **8 subtraction operations**
- **Method 3:** Convert to base 8 first

$$(25693)_{10} = (62135)_8 = (110\ 010\ 001\ 011\ 101)_2$$

Divisions successives par 8 => **5 division operations.**

- **Method 4:** Convert to base 16 first

(25693)10 = (645D)16 = (0110 0100 0101 1101)2

Successive divisions by 16=> **4 division operations**

Converting a large decimal number to binary

To convert a large decimal number to binary, it's recommended to initially convert it to **octal** or **hexadecimal**, and then proceed to binary conversion => to reduce the number of operations (division or subtraction).

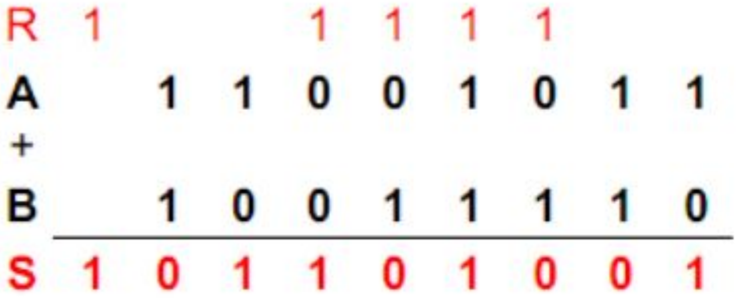
In general:

Method	**Successive divisions**	**Successive subtractions**	**Convert to base 8**	**Convert to base 16**
Number of Arithmetic Operations	Nombre des bits	Number of 1 bits	Number of bits per 3	Number of bits per 4

Binary Addition

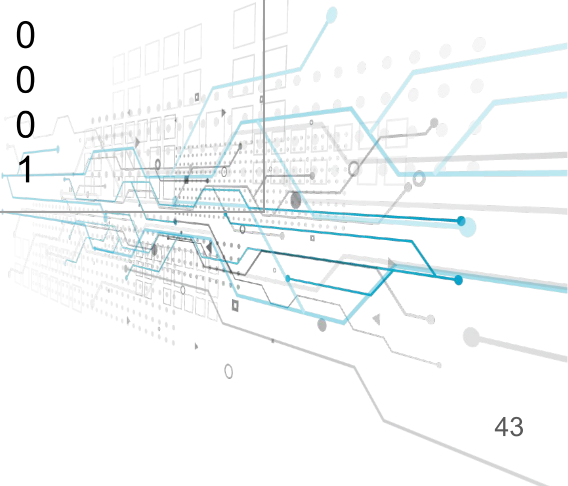
Whenever the result **exceeds 1**, it produces a **carryover** to the **adjacent column** with a **higher weight**.

A + B = 203 + 158 = 361



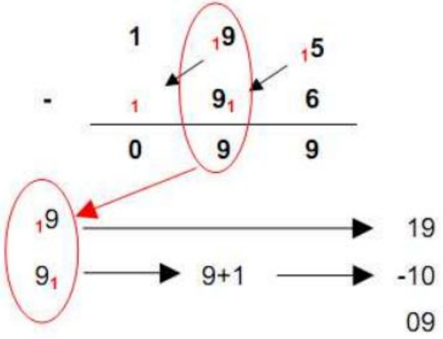
R	1			1	1	1	1		
A		1	1	0	0	1	0	1	1
+									
B		1	0	0	1	1	1	1	0
S	1	0	1	1	0	1	0	0	1

a	b	Sum	retainer
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

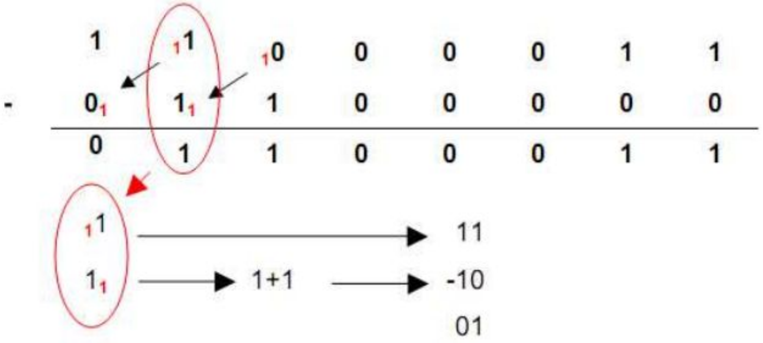


Binary Subtraction

If the digit being subtracted has a **lower** numerical value than the subtracting digit, we **borrow** from the the **column of higher weight.**



195 – 96 = 99



11000011 – 1100000 = 1100011

a	b	difference	retainer
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

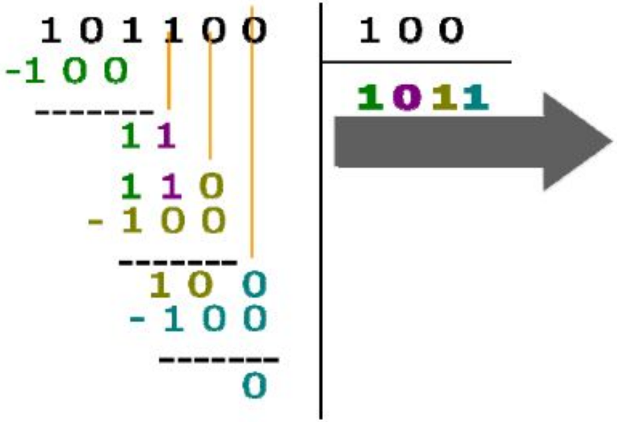
Binary Multiplication

Successive addition of the multiplican such that **multiplication by 0** results in a result equal to **0** and **multiplication by 1** results in the **copying of the multiplicand.**

				1	0	1	1
×					1	1	0
				$^{1}0$	0	0	0
		^{1}	$^{1}1$	0	1	1	.
	^{1}	1	0	1	1	.	.
	1	0	0	0	0	1	0

Binary Division

Successive subtraction of dividend divisor.



Arithmetic Operations in a numeral system with any base

Performing the four basic arithmetic operations in a numeral system with a base other than 10 or 2 can be challenging. In such cases, when we have two numbers N1 and N2 with different bases, B1 and B2 respectively, it's helpful to convert them to either decimal or binary before proceeding with the desired operation

Exercise:

We have two numbers **A** and **B** represented in three positions as follows:

$$\mathbf{A = (a_3\ a_2\ a_1)_5;\ B = (b_3\ b_2\ b_1)_7}$$

1. What are the possible values for the coefficients a_i, b_i?

2. Knowing that $\mathbf{A + B = (138)_9}$, $\mathbf{A - B = (200)_6}$, Find the values of the coefficients a_i, b_i.

3. Transform A and B into binary then calculate A+B, A-B, A * (B/100), A/B.

Solution:

1. $a_i < 5$ so, $a_i \in \{0, 1, 2, 3, 4\}$; $b_i < 7$ so, $b_i \in \{0, 1, 2, 3, 4, 5, 6\}$;

2. $$\begin{cases} A+B= \mathbf{(138)_9} \\ A-B= \mathbf{(200)_6} \end{cases}$$

The sum of these two equations gives : $2A= \mathbf{(138)_9} + \mathbf{(200)_6}$

You must convert the two operands to the same base to perform the operation.

The best choice is to convert them to a decimal base.

Solution:

$$(138)_9 = 1\times9^2 + 3\times9^1 + 8\times9^0 = (116)_{10}$$

$$(200)_6 = 2\times6^2 + 0\times6^1 + 0\times6^0 = (72)_{10}$$

$$2A = 116 + 72 \leftrightarrow A = (94)_{10}$$

$$B = 116 - A \leftrightarrow B = (22)_{10}$$

$$\left\{\begin{array}{l} A = (94)_{10} = (??)_5 \\ B = (22)_{10} = (??)_7 \end{array}\right.$$

By the method of successive divisions, we find:

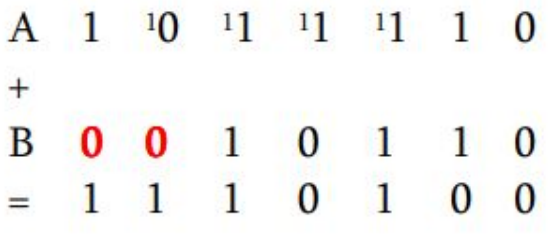
$$\left\{\begin{array}{l} A = (334)_5 \\ B = (031)_7 \end{array}\right.$$

By identification: $a_3 = 3$; $a_2 = 3$; $a_1 = 4$

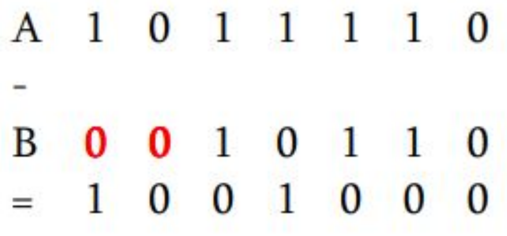
$b_3 = 0$; $b_2 = 3$; $b_1 = 1$

Solution:

3. $A = (94)_{10} = (1011110)_2$
$B = (22)_{10} = (10110)_2$



A	1	$^{1}0$	$^{1}1$	$^{1}1$	$^{1}1$	1	0
+							
B	0	0	1	0	1	1	0
=	1	1	1	0	1	0	0



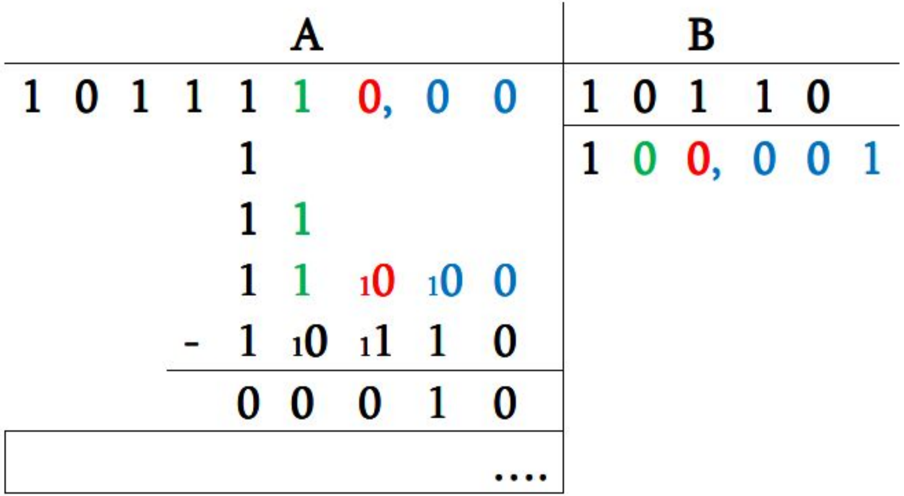
A	1	0	1	1	1	1	0
-							
B	0	0	1	0	1	1	0
=	1	0	0	1	0	0	0

Solution:

			B						
1	0	1	1	0.	0	1	0	0	
		1	1			1	0	1.	1
		1	1	0					
			1	0.	0				

A					1	0	1	1	1	1	0
*											
B/100								1	0	1,	1
=											
					$^{11}1$	$^{0}0$	$^{1}1$	$^{1}1$	1	1	0
+				$^{0}1$	0	1	1	1	1	0	.
+			$^{11}0$	0	0	0	0	0	0	.	.
+	^{1}	$^{1}1$	0	1	1	1	1	0	.	.	.
=	1	0	0	0	0	0	0	1	0	1,	0

Solution:



A	B
1 0 1 1 1 1 0, 0 0	1 0 1 1 0
1	1 0 0, 0 0 1
1 1	
1 1 $_{1}0$ $_{1}0$ 0	
- 1 $_{1}0$ $_{1}1$ 1 0	
0 0 0 1 0	
....	