TD 02

Exercise 1

Let A, B, $C \in P(E)$, proving the following :

1. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.
2. $A \subset B \Leftrightarrow A \cup B = B$
3. $A \cup B = A \cap C \Leftrightarrow B \subset A \subset C$

Exercise 2

Given A, B and C three parts of a set E,

1. Show that :
   (a) $(A \cap B) \cup \complement_E B = A \cup \complement_E B$.
   (b) $(A \setminus B) \setminus C = A \setminus (B \cup C)$.
2. Simplify :
   (a) $\overline{(A \cup B)} \cap \overline{(C \cup \overline{A})}$.
   (b) $\overline{(A \cap B)} \cup \overline{(C \cap \overline{A})}$.

Exercise 3

Let E, F two sets, $f : E \to F$ is an application, proving the following :

1. $\forall\ A, B \in P(E),\ f(A \cap B) \subset f(A) \cap f(B)$.
2. $\forall\ A, B \in P(E),\ f(A \cup B) = f(A) \cup f(B)$.
3. $\forall\ A \in P(F),\ f^{-1}\ (F \setminus A) = E \setminus f^{-1}(A)$.

Exercise 4

1. Describe the direct image of $\mathbb{R}$ by the exponential function
2. Find $f([0, 1[)$, $f(\mathbb{R})$, $f(]-1, 2[)$, $f^{-1}\ (\{3\})$, par la fonction $f : x \mapsto x^2$, defined on $\mathbb{R}$.

Exercise 5

Let $f : [1; +\infty[\to [0; +\infty[$ is a function such that $f(x) = x^2 - 1$ bijective ?

Exercise 6

Let E, F, G three sets, $f : E \to F$, $g : F \to G$ and $h : G \to E$

Proving that is $h \circ g \circ f$ is injective and if $g \circ f \circ h$ and $f \circ h \circ g$ are surjectives. Then, f , g and h are bijectives.