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Chapter 7: Transportation
Maximization Transportation Problem

In general, transportation model is used for cost minimization problems. However, it is also used to solve problems in which the objective is to maximize total value or benefit. That is, instead of unit cost c_{ij}, the unit profit or payoff p_{ij} associated with each route, (i, j) is given. The objective is to maximize the total profit for which the profit matrix is given. The objective is to maximize the total profit for which the profit matrix is given.

$$Maximize\ Z = \sum_{i=1}^{m} . \sum_{j=1}^{n} p_{ij} X_{ij}$$

Hence, when we would like to maximize the objective function. There are two methods:

a) The given matrix is to be multiplied by –1, so that the problem becomes maximization problem. Or b) Subtract all the elements in the matrix from the highest element in the matrix. Then the problem becomes maximization problem. Then onwards follow all the steps of maximization problem to get the solution.

Example 7: Four factories, A, B, C and D produce sugar and the capacity of each factory is given below: Factory A produces 10 tons of sugar and B produces 8 tons of sugar, C produces 5 tons of sugar and that of D is 6 tons of sugar. The sugar has demand in three markets X, Y and Z. The demand of market X is 7 tons, that of market Y is 12 tons and the demand of market Z is 4 tons. The following matrix gives the returns the factory can get, by selling the sugar in each market. Formulate a transportation problem and solve for maximizing the returns.

	Profit in DZ per ton (x 100) Markets			
factories	X	Y	Z	availability in tons
A	4	3	2	10
B	5	6	1	8
C	6	4	3	5
D	3	5	4	6
Requirement	7	12	3	**Σ b= 29, Σ b= 23**

Here Σ b is greater than Σ d hence we have to open a dummy column whose requirement constraint is 6, so that total of availability will be equal to the total demand. Now let get the basic feasible solution by VAM and then give optimality test by MODI method. The balanced matrix of the transportation problem is:

Row No.	X	Y	Z	Dummy	Availability	u_i
A	(2) [–4]	(3) [–3]	0 [–2]	(5) [–0]	10	0
B	2 [–5]	(8) [–6]	4 [–1]	3 [–0]	8	3
C	(5) [–6]	1 [–4]	2 [–3]	2 [–0]	5	2
D	3 [–3]	(1) [–5]	(5) [–4]	2 [–0]	6	2
Requirement	7	12	5	5	29	
column. no. v_j	4	3	2	0		

By multiplying the matrix by –1, we can convert it into a maximisation problem. Now in VAM we have to find the row opportunity cost and column opportunity costs. In minimisation problem, we use to subtract the smallest element in the row from next highest element in that row for finding row opportunity cost. Similarly, we use to subtract smallest element in the column by next highest elementin that column to get column opportunity cost. Here as we have multiplied the matrix by –1 the highest element will become lowest element. Hence subtract the lowest element from the next highest element as usual. Otherwise, instead of multiplying by –1 simply find the difference between highest element and the next lowest element and take it as opportunity cost of that row or column. For example in the given problem in the row A, the highest element is 4 and the next lowest element is 3 and hence the opportunity cost is 4 – 3 = 1. (Or smallest element is –4 and the next highest element is –3 and the opportunity cost is –3 – (– 4) = –3 + 4 = 1). Similarly, we can write all opportunity costs. Once we find the opportunity costs, rest of the procedure is same. That is, we have to select highest opportunity cost and select the highest profit element in that row or column for allocation. Obtain the basic feasible solution. As usual the basic feasible solution must have m + n – 1 allocations. If the allocations are not equal to m + n – 1, the problem degenerate. In that case, add ∈ to an empty cell, which do not form loop with other loaded cells. Once we have basic feasible solution, the optimality test by MODI method, is followed. Here, once the opportunity costs of all the cells are positive, (as we have converted the maximistion problem into minimisation problem) the solution is said to be optimal. In the given problem as the opportunity costs of all empty cells are positive, the solution is optimal. And the optimal return to the company is DA. 125/-. Allocations:

S.No	Loaded cell	Load	Cost in DA
1	AX	02	02*4= 08
2	AY	03	03*3= 09
3	A Dmy	05	05*0= 0
4	BY	08	08*6=48
5	CX	05	05*6= 30
6	DY	01	01*5= 05
7	DZ	05	05*4= 20
	Total returns		125

1.

	X	Y	Z	DMY		
A	4	3	2	0	10	(1)
B	5	6	1	0	8	(1)
						(2)
C	**5** 6	4	2	0	5	(2) ←
D	3	5	4	0	6	(1)
	7	12	5	5	29	
	1	1	2	0		

2.

	X	Y	Z	DMY		
A	4	3	2	0	10	1
B	5	6	1	0	8	1
D	3	5	4 **5**	0	6	1
	7	12	5	5	24	
	1	1	2 ↑	0		

3.

	X	Y	DMY		
A	4	3	0	10	1
B	5	6	0	8	1
D	3	5 **1**	0	1	2 ←
	2	12	5	19	
	1	1	0		

4.

	X	Y	DMY		
A	4	3	0	10	1
B	5	6 **8**	0	8	1
	2	11	5	18	
	1	3 ↑	0		

5.

	X	Y	DMY	
A	4 **2**	3 **3**	0 **5**	10
	2	3	5	10

Transportation problems (special cases)

1. Unbalanced transportation problem

The transportation problem wherein the total capacity of all sources and total requirement (demand) of all destinations are not equal is called the unbalanced transportation problem. An unbalanced transportation problem may occur in two different forms (i) Excess of availabilities (ii) Shortage in availabilities. In case

- **a.** we add a dummy destination in the transportation table with zero transportation cost. The requirement of this dummy destination is assumed to be equal to the difference of total availability of sources and total demand so that the problem becomes balanced.
- **b.** Similarly, in case b we introduce a dummy source and take the corresponding steps. The following example explain the whole procedure.

Example 8: A company has factories at A, B and C, which supply warehouses at D, E, F, and G. The monthly factory capacities are 160, 150 and 190 units, respectively. Monthly warehouse requirements are 80, 90, 110 and 160, respectively. Unit shopping costs (in Da) are as follows:

		To			
		D	E	F	G
From	A	42	48	38	37
	B	40	49	52	51
	C	39	38	40	43

Determine the optimum distribution for this company to minimize shipping costs.
Solution.
Here the total capacity of sources (factories) is 160 + 150 + 190 = 500. It is greater than the total requirement of all the destinations (warehouses) which is 80 + 90 + 110 + 160 = 440. Therefore, we added dummy destination in the transportation table with zero transportation cost and take 60 as its

requirement. Thus, the problem becomes balanced that is the total capacity and total requirement are equal the balanced, the problem is as follows:

		To					Capacity
		D	E	F	G	H	
From	A	42	48	38	37	0	160
	B	40	49	52	51	0	150
	C	39	38	40	43	0	190
Requiremnet		80	90	110	160	60	500

We can solve this problem using VAM to determine the basic feasible solution and then use MODI method to find the optimal solution. We have solved this problem in Example 3.

The optimum allocation is: 160 to (A, F), 80 to (B, D), 10 to (B, E), 60 to (B, G), 80 to (C, E) and 110 to (C, F).

2. Degenracy

In a transportation problem, if a basic feasible solution with m origins and n destinations has less than m + n -1 positive X_{ij} i.e. occupied cells, then the problem is said to be a degenerate transportation problem. The degeneracy problem does not cause any serious difficulty, but it can cause computational problem wile determining the optimal minimum solution. Therefore, it is important to identify a degenerate problem as early as beginning and take the necessary action to avoid any computational difficulty.

Example 9: Degeneracy occurs in Example 8. To solve this problem, we proceed as follows:

Solution: Using Vogel Approximation method, we get the basic feasible solution as:

Warehouse / Factory	D	E	F	G	H	Av.	D_1	D_2	D_3	D_4
A	42	48	38	**37** (160)	0 (e)	160	37	1	1	1
B	**40** (80)	49	**52** (10)	**51** (60)	0 (60)	150	40*	9	11*	1
C	39	**38** (90)	**40** (100)	43	0	190	38	1	3	3
Requirement	80	90	110	160	60	500				
$Diff_1$	1	10	2	6	0					
$Diff_2$	1	10*	2	6	-					
$Diff_3$	1	-	2	6	-					
$Diff_4$	-	-	2	6*	-					

Here the number of allocations is 6 but the number of allocations should be 3 + 5 – 1 = 7 to perform optimality test. Hence it is the case of degeneracy to resolve such a problem, we introduce infinitesimally small allocation e in the least cost and independent cell, i.e., the cell (A,H). Another cell (C, H) also has same least cost as the cell (A, H). But this cell is not independent because a loop (C,H)→ (C,F) → (B,F) → (B,H). Now the number of allocations is 7, i.e., as many as required for performing the optimality test.

42	48	38	●	0
●	49	● (B, F) →	51	● (B, H)
39	●	● (C, F) ←	43	0 (C, H)

Thus, applying optimality test by MODI method, the improved allocations after first iteration will be:

42	48	38 (e)	37 (160)	0
40 (80)	49	52 (10)	51	0 (60)
39	38 (90)	40 (100)	43	0

Again, applying the optimality test, the improved solution is

	D	E	F	G	H
A	42	48	38 (e)	37 (160)	0
B	40 (80)	49 (10)	52	51	0 (60)
C	39	38 (80)	40 (110)	43	0

Again, applying the optimality test, you will see that these allocations give the optimal solution and the total transportation cost = Da 17050.