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Chapter 7: Transportation

Methods of finding initial basic feasible solution

There are several methods to obtain initial basic feasible solution. Here, we shall discuss the following methods to determine the initial basic feasible solution:

1. North-West Corner Method (NWCM)

This method does not take into account the cost of transportation on any route of transportation. The method can be summarized as follows:

Step 1: Start with the cell at the upper left (north-west) corner of the transportation table (or matrix) and allocate commodity equal to the minimum of the rim values for the first row and first column, i.e. min (a1, b1). When total demand equals total supply, the transportation problem is said to be balanced. 260 Operations Research: Theory and Applications.

Step 2: (a) If allocation made in Step 1 is equal to the supply available at first source (a1, in first row), then move vertically down to the cell (2, 1), i.e., second row and first column. Apply Step 1 again, for next allocation. (b) If allocation made in Step 1 is equal to the demand of the first destination (b1 in first column), then move horizontally to the cell (1, 2), i.e., first row and second column. Apply Step 1 again for next allocation. (c) If a1 = b1, allocate x11 = a1 or b1 and move diagonally to the cell (2, 2).

Step 3: Continue the procedure step by step till an allocation is made in the south-east corner cell of the transportation table. Remark If during the process of making allocation at a particular cell, the supply equals demand, then the next allocation of magnitude zero can be made in a cell either in the next row or column. This condition is known as degeneracy.

Example 2: A company has three production facilities S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 and D_4 with requirement of 5, 6, 7 and 14 units (in 100s) per week, respectively. The transportation costs (DZ) per unit between factories to warehouses are given in the table below:

	D_1	D_2	D_3	D_4	Supply (Availability)
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Use North-West Corner Method (NWCM) to find an initial basic feasible solution to the transportation problem using data of table above.

Solution

The cell (S_1, D_1) is the north-west corner cell in the given transportation table. The rim values for row S_1 and column D_1 are compared. The smaller of the two, i.e. 5, is assigned as the first allocation; otherwise it will violate the feasibility condition. This means that 5 units of a commodity are to be transported from source S_1 to destination D_1. However, this allocation leaves a supply of 7 – 5 = 2 units of commodity at S1. Move horizontally and allocate as much as possible to cell (S_1, D_2). The rim value for row S_1 is 2 and for column D_2 is 8. The smaller of the two, i.e. 2, is placed in the cell. Proceeding to row S_2, since the demand of D_1 is fullfilled. The unfulfilled demand of D_2 is now 8 –

2 = 6 units. This can be fulfilled by S_2 with capacity of 9 units. So 6 units are allocated to cell (S_2, D_2). The demand of D_2 is now satisfied and a balance of 9 – 6 = 3 units remains with S_2.

	D_1	D_2	D_3	D_4	Supply
S_1	19 $\boxed{5}$	30 $\boxed{2}$	50	10	7
S_2	70	30 $\boxed{6}$	40 $\boxed{3}$	60	9
S_3	40	8	70 $\boxed{4}$	20 $\boxed{14}$	18
Demand	5	8	7	14	34

Continue to move horizontally and vertically in the same manner to make desired allocations. Once the procedure is over, count the number of positive allocations. These allocations (occupied cells) should be equal to m + n – l = 3 + 4 – l = 6. If yes, then solution is non-degenerate feasible solution. Otherwise degenerate solution. The total transportation cost of the initial solution is obtained by multiplying the quantity x_{ij} in the occupied cells with the corresponding unit cost c_{ij} and adding all the values together. Thus, the total transportation cost of this solution is Total cost = 5 × 19 + 2 × 30 + 6 × 30 + 3 × 40 + 4 × 70 + 14 × 20 = 1,015 DZ

2. Least Cost Method (LCM)

This method is also known as the Matrix Minimum method or Inspection method. It starts by making the first allocation to the cell for which the transportation cost per unit is lowest. The row or column for which the capacity is exhausted or requirement is satisfied is removed from the transportation table. We follow the procedure with the reduced matrix until all the requirements are satisfied. If there is a tie for the lowest cost cell while making any allocation, the choice may be made for a row or a column by which maximum requirement is exhausted. If there is a tie in making this allocation as well, then we can arbitrarily choose a cell for allocation.

Example 3: Find the basic feasible solution of the given transportation problem by applying Least Cost Method.

Factory \ Warehouse	D	E	F	G	capacity
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Requirement	80	90	110	220	500

Solution.

Here, the least cost is 37 in the cell (A, G). The requirement of the Warehouse G is 220 and the capacity of Factory A is 160. Hence, the maximum number of units that can be allocated to this cell is 160. Thus, Factory A is exhausted. The requirement of Warehouse G is reduced by 160. Now, the least cost is 38, which is in the cell (C, E). The requirement of the Warehouse E is 90 and the capacity of Factory C is 190. Hence, the maximum number of units that can be allocated to this cell is 90. Moreover, we reduce the capacity of factory C by 90. The least cost in the matrix is 39, which is in the cell (C, D). The requirement of Warehouse D is 80 and the capacity of Factory C is 100.Hence, the maximum number of units that can be allocated to this cell is 80. The requirement of Warehouse D is exhausted. The capacity of the Factory C is also reduced by 80. The least cost in this matrix is 40 which is in the cell (C, F). The requirement of Warehouse F is 110 and the capacity of Factory C is 20. Hence, the maximum number of units that can be allocated to this cell is 20. Thus, Factory C is exhausted. The requirement of Warehouse F is reduced by 20. It is now 90 in the reduced matrix. The least cost is 51 in the cell (B, G) and the requirement of warehouse G is 60 units. So, we allocate 60 units to cell (B, G) and the remaining 90 units to the cell (B, F). Thus, the allocations given using Least Cost method are as shown in the following matrix along with the cost per unit of transportation:

Warehouse / Factory	D	E	F	G	Capacity
A	42	48	38	37 $\boxed{160}$	160
B	40	49	52 $\boxed{90}$	51 $\boxed{60}$	150
C	39 $\boxed{39}$	38 $\boxed{90}$	40 $\boxed{20}$	43	190
Requirement	80	90	110	220	500

Thus, the total transportation cost = 37× 160 + 52 × 90 + 51 × 80 + 38 × 90 + 40 × 20 = 21000

Note that the minimum transportation cost obtained by the least cost method is much lower than the corresponding cost of the solution developed by using the north-west corner method.

3. Vogel's Approximation Method (VAM) (Penalty or Regret Method)

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: For each row (column) with strictly positive capacity (requirement), determine a penalty by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step 2: Identify the row or column with the largest penalty among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.

Step 3: We select X_{ij} as a basic variable if C_{ij} is the minimum cost in the row or column with largest penalty. We choose the numerical value of X_{ij} as high as possible subject to the row and the column constraints. Depending upon whether a_i or b_j is the smaller of the two i^{th} row or j^{th} column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

Example 4: Use Vagel's Approximation Method (VAM) to find the initial basic feasible solution to the transportation problem using the data of example 1.

Solution: The differences (penalty costs) for each row and column have been calculated as shown in Table below. In the first round, the maximum penalty, 22 occurs in column D_2. Thus the cell (S_3, D_2) having the least transportation cost is chosen for allocation. The maximum possible allocation in this cell is 8 units and it satisfies demand in column D_2. Adjust the supply of S_3 from 18 to 10 (18 – 8 = 10).

	D_1	D_2	D_3	D_4	Supply	Row differences
S_1	19 $\boxed{5}$	30	50	10 $\boxed{2}$	7	9 9 40 40
S_2	70	30	40 $\boxed{7}$	60 $\boxed{2}$	9	10 20 20 20
S_3	40	8 $\boxed{8}$	70	20 $\boxed{10}$	18	12 20 50 -
Demand	5	8	7	14	34	
Column differences	21 21 - -	22 - - -	10 10 10 10	10 10 10 50		

The new row and column penalties are calculated except column D_2 because D_2's demand has been satisfied. In the second round, the largest penalty, 21 appears at column D_1. Thus the cell (S_1, D_1) having the least transportation cost is chosen for allocating 5 units as shown in Table above. After adjusting the supply and demand in the table, we move to the third round of penalty calculations. In the third round, the maximum penalty 50 appears at row S_3. The maximum possible allocation of 10 units is made in cell (S_3, D_4) that has the least transportation cost of 20 as shown in Table above. The process is continued with new allocations till a complete solution is obtained. The initial solution using VAM is shown in Table above. The total transportation cost associated with this method is: Total cost = $5 \times 19 + 2 \times 10 + 7 \times 40 + 2 \times 60 + 8 \times 8 + 10 \times 20 = 779$ DZ