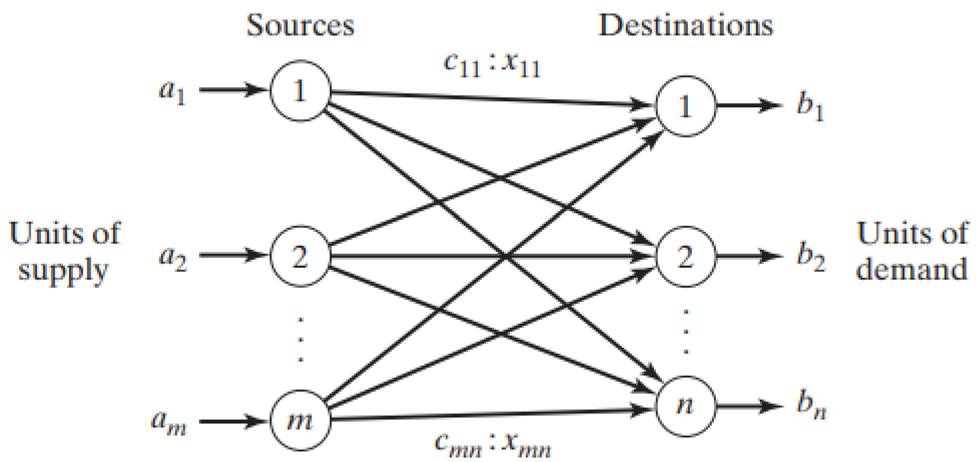
Prepared by: Dr. Lemya Mekarssi. **Level: 2nd Year Bachelor of Business Sciences**

Chapter 7: Transportation

Transportation problem is a special kind of Linear Programming Problem (LPP) in which the objective is to transport goods from a set of sources/origins to a set of destinations in such a manner that the total transportation or shipping cost is minimized. To achieve this objective, we must know about some parameters such as the quantity of available supplies, the quantity demanded and the costs of shipping a unit from various origins to various destinations

Definition of the transportation model

The problem is represented by the network in Figure below. There are m sources and n destinations, each represented by a **node**. The **arcs** represent the routes linking the sources and the destinations. Arc (i, j) joining source i to destination j carries two pieces of information: the transportation cost per unit, Cij, and the amount shipped, Xij. The amount of supply at source i is a_i, and the amount of demand at destination j is bj. The objective of the model is to minimize the total transportation cost while satisfying all the supply and demand restrictions.



Mathematical formulation of the transportation problem

Let there be m origins/ sources of supply $O1,2, \dots , Oi \dots Om$ and n destinations $D1,D2, \dots ,Dj \dots Dn$. The total number of the capacities of all m origins is assumed to be equal to the total number of the requirements of all n destinations. Let Cij be the cost of shipping one unit from origin i to destination j. Let ai be the capacity/ availability of items at origin i and b , the requirement/demand of the destination j. Then this transportation problem can be expressed in a tabular form as follows: The condition for the existence of a feasible solution to a transportation problem is give as

Origin	Destinations						Availability/ capacity
	D_1	D_1	...	D_j	...	D_n	
O_1	C_{11}	C_{12}	...	C_{1j}	...	C_{1n}	a_1
O_2	C_{21}	C_{22}	...	C_{2j}	...	C_{2j}	a_2
	.	.		.		.	.
O_i	C_{i1}	C_{i2}	...	C_{ij}	...	C_{in}	a_i
	.	.		.		.	.
O_m	C_{m1}	C_{m2}	...	C_{mj}	...	C_{mn}	a_m
Requirement/ Demand	b_1	b_2	...	b_j	...	b_n	Total

$$\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$$

The above equation tells us that the total requirement/demand equals the total capacity. If it is not so, a dummy origin or destination is created to balance the total capacity and requirement. Now let x_{ij} be the number of units to be transported from origin i to destination j and C_{ij} the corresponding cost of transportation. Then the total transportation cost is :

$$\boxed{\sum_{i=1}^{m} \cdot \sum_{j=1}^{n} C_{ij} X_{ij}}$$

Subject to the constraints :

$$\sum_{j=1}^{n} X_{1j} = a_1, \sum_{j=1}^{n} X_{2j} = a_2, \dots, \sum_{j=1}^{n} X_{mj} = a_m$$

$$\sum_{i=1}^{m} X_{i1} = b_1, \sum_{i=1}^{m} X_{i2} = b_2, \dots, \sum_{i=1}^{m} X_{in} = b_n$$

$X_{ij} \geq 0$, for all i= 1,2,3…m and j=1,2…n

In the transportation method, we first obtain the initial basic feasible solution and then perform the optimality test.

Example 1: Suppose a manufacturing company owns three factories (sources) and distribute his products to five different retail agencies (destinations). The following table shows the capacities of the three factories, the quantity of products required by the various retail agencies and the cost of shipping one unit of the product from each of the three factories to each of the five retail agencies.

	Retail Agency					
factories	1	2	3	4	5	capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Usually the above table is referred as Transportation Table, which provides the basic information regarding the transportation problem. The quantities inside the table are known as transportation cost per unit of product. The capacity of the factories 1, 2, 3 is 50, 100 and 150 respectively. The requirement of the retail agency 1, 2, 3, 4, 5 is 100,60,50,50, and 40 respectively.

In this case, the transportation cost of one unit
From factory 1 to retail agency 1 is 1,
From factory 1 to retail agency 2 is 9,
From factory 1 to retail agency 3 is 13, and so on.
A transportation problem can be formulated as linear programming problem using variables with two subscripts. Let
X_{11}=Amount to be transported from factory 1 to retail agency 1
X_{12}= Amount to be transported from factory 1 to retail agency 2
.
.
.
X_{35}= Amount to be transported from factory 3 to retail agency 5.
Let the transportation cost per unit be represented by C_{11}, C_{12}, …..C_{35} that is $C_{11}=1$, $C_{12}=9$, and so on. Let the capacities of the three factories be represented by $a_1=50$, $a_2=100$, $a_3=150$. Let the requirement of the retail agencies are $b_1=100$, $b_2=60$, $b_3=50$, $b_4=50$, and $b_5=40$.
Thus, the problem can be formulated as
Minimize $C_{11}x_{11}+C_{12}x_{12}+\ldots\ldots\ldots\ldots+C_{35}x_{35}$
Subject to:
$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = a_1$
$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = a_2$
$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = a_3$
$x_{11} + x_{21} + x_{31} = b_1$
$x_{12} + x_{22} + x_{32} = b_2$
$x_{13} + x_{23} + x_{33} = b_3$
$x_{14} + x_{24} + x_{34} = b_4$
$x_{15} + x_{25} + x_{35} = b_5$
$x_{11}, x_{12}, \ldots\ldots, x_{35} \geq 0$.
Thus, the problem has 8 constraints and 15 variables. So, it is not possible to solve such a problem using simplex method. This is the reason for the need of special computational procedure to solve transportation problem.

The transportation Algorithm

The algorithm for solving a transportation problem may be summarized into the following steps:
Step 1: Formulate the problem and arrange the data in the matrix form: The formulation of the transportation problem is similar to the LP problem formulation. In transportation problem, the objective function is the total transportation cost and the constraints are the amount of supply and demand available at each source and destination, respectively.

Step 2: Obtain an initial basic feasible solution, following three different methods are discussed to obtain an initial solution:

• North-West Corner Method;

• Least Cost Method;

• Vogel's Approximation (or Penalty) Method.

The initial solution obtained by any of the three methods must satisfy the following conditions:

- ✓ The solution must be feasible, i.e. it must satisfy all the supply and demand constraints (also called rim conditions).

- ✓ The number of positive allocations must be equal to m + n – 1, where m is the number of rows and n is the number of columns. Any solution that satisfies the above conditions is called non-degenerate basic feasible solution, otherwise, degenerate solution.

Step 3: Test the initial solution for optimality, the Modified Distribution (MODI) method is discussed to test the optimality of the solution obtained in Step 2. If the current solution is optimal, then stop. Otherwise, determine a new improved solution.

Step 4: Updating the solution Repeat Step 3 until an optimal solution is reached.