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Chapter 4: Linear Programming: The Simplex Method

The Simplex Method with Mixed Constraints

Previously, we discussed how to convert inequalities (greater than or equal to, or less than or equal to) into equations to apply the simplex method. Since the mixed formulation can also include constraints in the form of equations, in this case, an artificial variable Ai is added without the need to add a slack variable because the right-hand side equals the left-hand side. It is written according to the following formula:

$$ax + A_i = b_i$$

This artificial variable is also added to the objective function with a very large coefficient] M [and a positive sign in case of minimization (and a negative sign in case of maximization.

Example: Solve the following linear program using the simplex method

$$\text{Min : } Z = 5X_1 + 7X_2$$
$$X_1 \leq 20$$
$$X_2 \geq 20$$
$$X_1 + 2X_2 = 50$$
$$X_1 \geq 0,\ X_2 \geq 0$$

Solution:

1. Converting the program to the standard form:

$$\text{Min: } Z = 5X_1 + 7X_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$
$$X_1 + \quad + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 20$$
$$+X_2 + 0S_1 - 1S_2 + 1A_1 + 0A_2 = 20$$
$$X_1 + 2X_2 + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 50$$
$$X_1 \geq 0,\ X_2 \geq 0,\ S_1 \geq 0,\ S_2 \geq 0\ A_1 \geq 0,\ A_2 \geq 0$$

2. Obtaining the Initial Solution:

Initial Simplex Table

C_b		5	7	0	0	M	M	B_i	B_i/a_{ij}
		X_1	X_2	S_1	S_2	A_1	A_2		
0	S_1	1	0	1	0	0	0	20	-
M	A_1	0	1	0	-1	1	0	20	20/1=20
M	A_2	1	2	0	0	0	1	50	50/2=25
$Z_j = \sum_{j=1}^{n} C_j X_j$		M	3M	0	-M	M	M	Z=20M+50M	
$C_j - Z_j$		5-M	7-3M	0	M	0	0		

Pivot element

-The incoming variable is: x_2, the outgoing variable is A_1.

3. Evaluate the possibility of improving the solution

Referring to the values of the last line, we note that there are two negative values and this means that the optimal solution has not yet been reached.

Second Simplex Table

C_b		5	7	0	0	M	M	B_i	B_i/a_{ij}
		X_1	X_2	S_1	S_2	A_1	A_2		
0	S_1	1	0	1	0	0	0	20	/
7	X_2	0	1	0	-1	1	0	20	/
M	A_2	1	0	0	2	0	1	50	50/2=25
$Z_j = \sum_{j=1}^{n} C_j X_j$		**M**	**7**	**0**	**-7+2M**	**7-2M**	**M**	**Z=140+10M**	
C_j-Z_j		**5-M**	**0**	**0**	**7-2M**	**-7+2M**	**0**		

Pivot element

-The incoming variable is: s_1, the outgoing variable is A_2.

Referring to the values of the last line, we note that there are one negative value and this means that the optimal solution has not yet been reached.

Optimal Simplex Table

C_b		5	7	0	0	M	M	B_i	B_i/a_{ij}
		X_1	X_2	S_1	S_2	A_1	A_2		
0	S_1	1	0	1	0	0	0	20	-
7	X_2	1/2	1	0	0	-1	1/2	25	-
M	S_2	1/2	0	0	1	0	1/2	5	-
$Z_j = \sum_{j=1}^{n} C_j X_j$		**7/2**	**7**	**0**	**0**	**0**	**7/2**	**Z=175**	
C_j-Z_j		**3/2**	**0**	**0**	**0**	**M**	**M-7/2**		

It is clear from the values of the last line that some values are positive and some are equal to zero and therefore the optimal solution has been reached and the third table is the optimal table. Whereas, the values of the variables:

- Basic Variables: S_1= 20, X_2= 25, S_{12}= 5

- Non-basic variables: A_1=0, A_2=0, X_1=0

To be sure, we substitute in the objective function: Z=5(0) +7 (25)=175