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Chapter 3: Linear Programming: Graphical Method

3. Special cases of the graphical method:

a. Multiple optimal solutions: In some cases, we find more than one optimal solution, that is, the values of the objective function Z are equal.

Example

$$\text{Max: } Z=2X_1+4X_2$$
$$X_1 \leq 8$$
$$X_2 \leq 3$$
$$3X_1+6X_2 \leq 30$$
$$X_1 \geq 0,\ X_2 \geq 0$$

Solution:

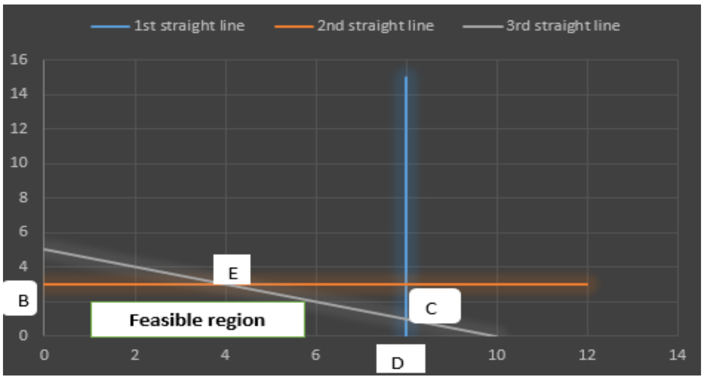
- Converting the inequalities forming the constraints of the problem into equations as follows:

$$X_1 = 8$$
$$X_2 = 3$$
$$3X_1+6X_2=30$$

3rd straight line: $3X_1+6X_2= 30$		
	X_1	**X_2**
point 1	0	5
point 2	10	0
	After drawing the straight line, we shade the region that achieves the inequality solutions for the third constraint, which are all the points under the line.	

Feasible solutions to the LP:

corner points			Z
	X_1	X_2	
A	0	0	0
B	0	3	12
C	8	1	20
E	4	3	20
D	8	0	16



We note that the feasible region is confined to the area (A, B, E, C, D), and through the results obtained, the optimal solution is achieved at the two points E, C, which represent the greatest possible profit, and here the company can choose the best and most appropriate decision for it.

2. Lack of solutions: In this case, the feasible region cannot be formed, which requires modification of the model.

Example

$$\text{Max: } Z=2X_1+5X_2$$

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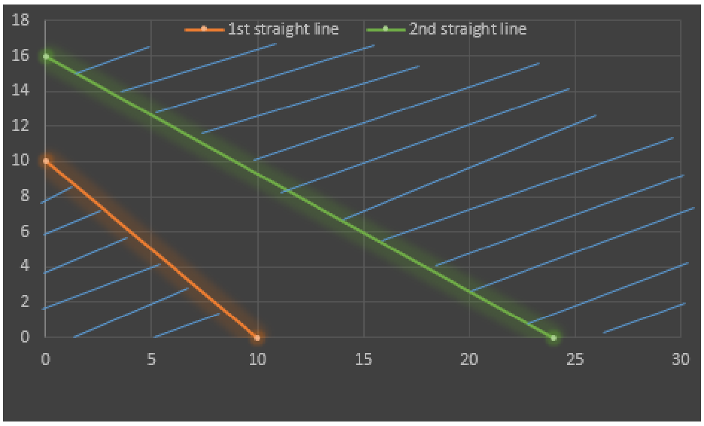
$$\begin{cases} X_1+X_2 \leq 10 \\ 2X_1+3X_2 \geq 48 \\ X_1 \geq 0,\ X_2 \geq 0 \end{cases}$$

Solution :

1st straight line: $X_1+X_2=10$			**2nd straight line: $2X_1+3X_2=48$**		
	X_1	**X_2**		**X_1**	**X_2**
point 1	0	10	point 1	0	16
point 2	10	0	point 1	24	0
	No possible solutions area				



We note from the figure that there is no area of possible solutions, i.e. no optimal solution, which necessitates reconsidering the model, whether by adding other restrictions or correcting the previous restrictions, so that the area of possible solutions is formed.

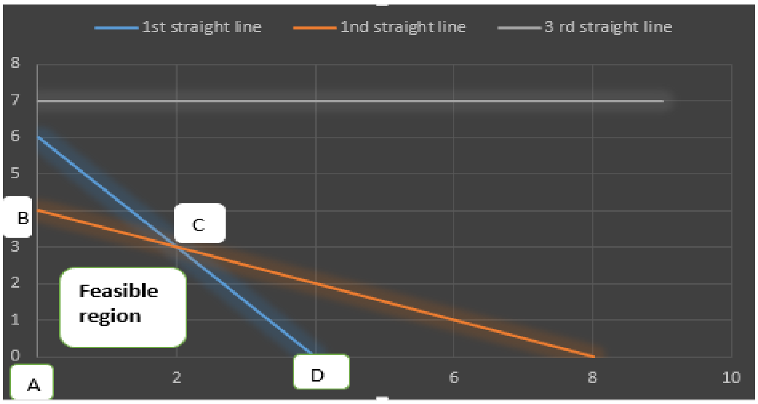
3. The case that one of the constraints (conditions) does not affect the feasible region: In this case, we find that one of the constraints is far from the feasible region, that is, it does not affect the optimal solution, in the sense that it can be deleted from the form.

Example

$$\text{Max : } Z=12X_1+8X_2$$
$$6X_1+4X_2\leq 24$$
$$2X_1+4X_2\leq 16$$
$$X_2\leq 7$$
$$X_1\geq 0,\ X_2\geq 0$$

Solution :

1st straight line: $6X_1+4X_2=24$			**2nd straight line: $2X_1+4X_2=16$**		
	X_1	**X_2**		**X_1**	**X_2**
point 1	0	6	point 1	0	4
point 2	4	0	point 2	8	0

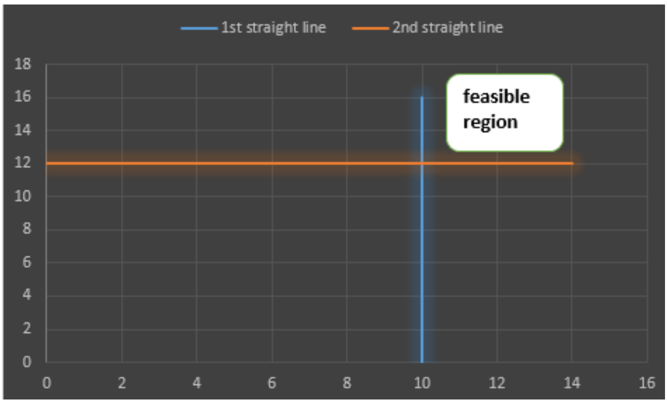


We note from the figure that the area of possible solutions is defined by the lines 1 and 2 and is confined to the area (A, B, C, D). It did not change when we added constraint 3, that is, constraint 3 did not affect the area of possible solutions. Thus it can be excluded from the mathematical model and solve the model with only two equations.

4. Case of an infinite number of solutions (infinite objective function): In this case, the objective function increases indefinitely without affecting the constraints.

Example:

$$\text{Max : } Z=6X_1+4X_2$$
$$X_1\geq 10$$
$$X_2\geq 12$$
$$X_1\geq 0,\ X_2\geq 0$$



We note from the figure that the solution area goes to infinity, that is, the existence of an infinite number of solutions, which requires referring to the mathematical model and re-correcting or modifying it.