Exercise series No 2

Note: ***questions marked * left to the students***

Exercise 1

1) Show that the function $y = x + 1 - \frac{1}{3}e^{x}$, is a solution to the first-order initial value problem $\frac{dy}{dx} = y - x;\ y(0) = \frac{2}{3}$, on $\mathbb{R}$.

2)* Show that every member of the family of functions $y = \frac{C}{x} + 2$, is a solution of the first-order differential equation $\frac{dy}{dx} = \frac{1}{x}(2 - y)$ on the interval $]0, +\infty[$, where C is any constant.

3) give a differential equation whose solutions are of the form:

$$y = \frac{c + x}{x^2 + 1};\quad c \in \mathbb{R}.$$

Exercise 2

Solve the following separable differential equations:

a)* $y' + 4y = 0;\ y(0) = 2.$

b) $2x + yy' = 0; y(1) = 1.$

c) $xy' + (1 + x)y = 0;\ y(1) = 1.$

d)* $(1 + x^2)y' - xy = 0;\ \ y(0) = 1.$

e) $(4 - x^2)yy' = 2(1 + y^2).$

f)* $(1 + y)y' = 4x^3;\ \ ; y(0) = 0.$

Exercise 3

Solve the following linear differential equations:

a) $xy' + y = x;\ y(2) = 0.$

b)* $y' + y = xe^{x}; y(0) = 1.$

c) $y' - 2y = -\frac{2}{1+e^{-2x}};\ y(0) = 2.$

d)* $y' + \tan x\, y = \sin 2x;\ \ y(0) = 1; x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[.$

e)* $(1 + x)y' + xy = x^2 - x + 1;\ y(1) = 1.$

Exercise 4

Solve the following Bernoulli's differential equation:

a) $xy' + 3y = x^2y^2;$

b)* $y' + y\cot x + y^2 = 0.$

c) $y' + 2xy = -xy^4.$