Exercise series No 1

Note: ***questions marked * left to the students***

Exercise 01

Calculate the *lower Darboux sum* $s(d, f)$ and the *upper Darboux sum* $S(d, f)$ of the function f, attached to the equal-step division over the domain [a, b] in each of the following cases and then conclude that f is Riemann integrable in this domain:

a) $[a, b] = [1,2]\ ;\ f(x) = \frac{1}{x}$ b)* $[a, b] = [2,5]\ ;\ f(x) = lnx$

c)* $[a, b] = \left[\frac{\pi}{2}, \pi\right]\ ;\ f(x) = cosx.$

Exercise 02

Using the Riemann sum of an appropriate function, determine, in each of the following cases, the limit of the sequence $(u_n)_{n\in\mathbb{N}^*}$.

a) $u_n = \sum_{k=0}^{n} \frac{n}{(n+k)^2}$ b)* $u_n = \sum_{k=1}^{n} \frac{1}{\sqrt{n}\sqrt{n+k}}$ c)* $u_n = \prod_{k=0}^{n} \left(1 + \frac{k}{n}\right)^{\frac{1}{n}}.$

Exercise 03

Using integration by changing the variable, calculate the following integrals:

a) $\int \frac{1}{3\sqrt[3]{x+1} - x + 1} dx$ b) $\int_{-1}^{\frac{1}{2}} \sqrt{x^2 + 2x + 5}\, dx$ c) $\int \frac{\sin x}{1 + \sin x} dx.$

d)* $\int \sqrt{-x^2 + 2x + 3}\, dx$ e)* $\int_0^{\frac{\pi}{2}} \frac{cos^3 x}{\sqrt{1 + sinx}} dx$ f)* $\int_0^1 x\sqrt{\frac{x}{x+2}}\, dx.$

Exercise 04

Using integration by parts, calculate the following integrals:

a) $\int x^2 ln\frac{x-1}{x} dx$ b)* $\int_0^1 xArc\tan x\, dx$ c)* $\int x^2 e^{2x}\, dx$

d) $\int_0^{\frac{\pi}{2}} \cos 2x \sin x\, dx.$

Exercise 05

Calculate the following integrals:

a) $\int \frac{x^5 + 3x^4 + 3x}{(x^2+1)(x+2)^2} dx$ b) $\int_1^3 \sqrt{x} ln\frac{x+1}{x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 + \cos x + \sin x} dx$

d)* $\int_0^1 xArc\tan\frac{x+1}{x}\,dx$ e)* $\int \frac{1}{3x-5}\sqrt{\frac{x+1}{x-1}}\,dx$ f)* $\int \frac{15\sin x}{8-10\sin x}\,dx$.

Exercise 06

Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by: $f(x) = \begin{cases} \sqrt{x}\ln(x+1), & x > 0 \\ \sqrt{x^2-2x}, & x \leq 0 \end{cases}$

a) Show that f is continuous on $\mathbb{R}$.

b) Calculate the integral $\int_{-1}^{1} f(x)\,dx$.