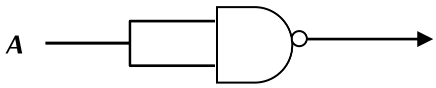
<u>Solutions of the series No. 01</u>

<u>Exercise 1 :</u>

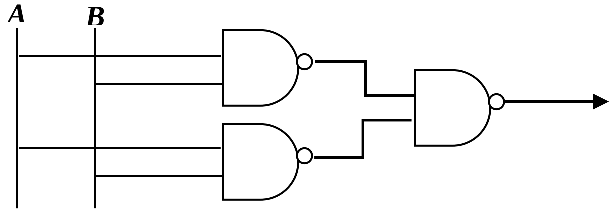
1. Writing and representation of logical expressions ($\bar{A}$, $A.B$, $A+B$)

A. Using **" NAND "** only

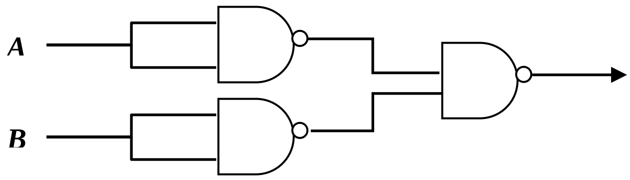
- $\bar{A} = \overline{A.A}$



- $A.B = \overline{\overline{A.B}}$
$= \overline{\overline{A.B}\ .\ \overline{A.B}}$

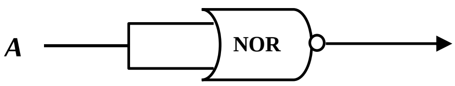


- $A+B = \overline{\overline{A+B}}$
$= \overline{\bar{A}\ .\ \bar{B}}$
$= \overline{\overline{A\ .\ A}\ .\ \overline{B\ .\ B}}$

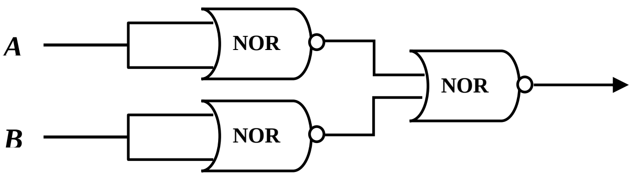


B. Using **“ NOR ”** only

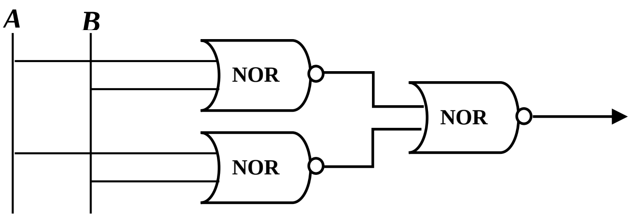
- $\bar{A} = \overline{A+A}$



- $A\ .\ B = \overline{\overline{A\ .\ B}}$
$= \overline{\bar{A}\ +\ \bar{B}}$
$= \overline{\overline{A+A}\ +\ \overline{B+B}}$



- $A+B = \overline{\overline{A+B}}$
$= \overline{\overline{A+B}\ +\ \overline{A+B}}$

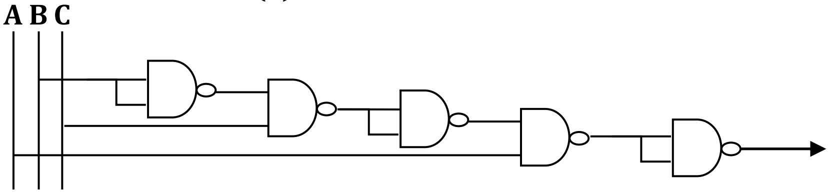


2. Writing the formula $A.\overline{(B+\bar{C})}$ using **“ NAND ”** only:

$$A.\overline{(B+\bar{C})} = \overline{\overline{A.\overline{(B+\bar{C})}}} = \overline{\overline{A.\overline{(B+\bar{C})}} \cdot \overline{A.\overline{(B+\bar{C})}}} \quad \Lambda\ (1)$$

$$\overline{(B+\bar{C})} = \bar{B}\cdot C = \overline{\overline{\bar{B}\cdot C}} = \overline{\overline{\bar{B}\cdot C}\cdot\overline{\bar{B}\cdot C}} = \overline{\overline{\overline{B.B}\cdot C}\cdot\overline{\overline{B.B}\cdot C}}$$

$$\Rightarrow\ (1) = \overline{\overline{A.\overline{\overline{\overline{B.B}\cdot C}\cdot\overline{\overline{B.B}\cdot C}}}\ .\ \overline{A.\overline{\overline{\overline{B.B}\cdot C}\cdot\overline{\overline{B.B}\cdot C}}}}$$



Exercise 2:

$$F = \big(A \odot (A \odot B)\big) \odot \big(B \odot (A \odot B)\big)$$

$$= \overline{\overline{A \oplus \big(\overline{A \oplus B}\big)} \oplus \overline{B \oplus \big(\overline{A \oplus B}\big)}}$$

$$= \overline{\overline{A \oplus (A.B + \bar{A}.\bar{B})} \oplus \overline{B \oplus (A.B + \bar{A}.\bar{B})}}$$

$$= \overline{\big((A.(A.B + \bar{A}.\bar{B}) + \bar{A}.\overline{(A.B + \bar{A}.\bar{B})}\ \big) \oplus \big(B.(A.B + \bar{A}.\bar{B}) +\ \bar{B}.\overline{(A.B + \bar{A}.\bar{B})}\ \big)}$$

$$= \overline{\big((A.(A.B + \bar{A}.\bar{B}) + \bar{A}.(A.\bar{B} + \bar{A}.B)\big) \oplus (B.(A.B + \bar{A}.\bar{B}) +\ \bar{B}.(A.\bar{B} + \bar{A}.B)\ )}$$

$$= \overline{\big(((A.A.B + A.\bar{A}.\bar{B}) + (\bar{A}.A.\bar{B} + \bar{A}.\bar{A}.B)\big) \oplus \big((B.A.B + B.\bar{A}.\bar{B}) +\ (\bar{B}.A.\bar{B} + \bar{B}.\bar{A}.B)\ \big)}$$

$$= \overline{\big(((A.B + 0) + (0 + \bar{A}.B)\big) \oplus \big((A.B + 0) +\ (A.\bar{B} + 0)\ \big)}$$

$$= \overline{(A.B + \bar{A}.B) \oplus (A.B + A.\bar{B}\ )}$$

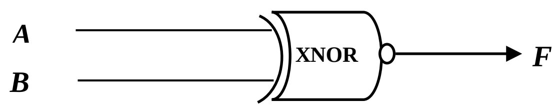
$$= \overline{\big(B.(A + \bar{A}\ )\big) \oplus \big(A.(B + \bar{B}\ )\big)}$$

$$= \overline{\big(B.(1)\big) \oplus \big(A.(1)\big)}$$

$$= \overline{(B) \oplus (A)}$$

$$= \overline{A \oplus B}$$

$$= A \odot B$$



Exercise 3:

A. Normal expressions and representations (without NAND and NOR)

1) Determination of input and output variables:

- We have 03 input variables representing the 03 switches which are represented by A, B, and C respectively.
- We have 02 output variables representing the 02 lamps which are represented by L1 and L2 respectively.

2) Truth table:

- **Variables:** I1, I2 and I3 binary (boolean) variables→ **1** closed switch,
  → **0** open switches.
- **Functions:** L1 and L2 binary functions (boolean)→ **1** lamp works,
  → **0** lamp does not work.

I1	I2	I3	**L1**	**L2**
0	0	0	**0**	**0**
0	0	1	**1**	**0**
0	1	0	**1**	**0**
0	1	1	**1**	**1**
1	0	0	**1**	**0**
1	0	1	**1**	**1**
1	1	0	**1**	**1**
1	1	1	**1**	**1**

1) **Simplification:**

L1

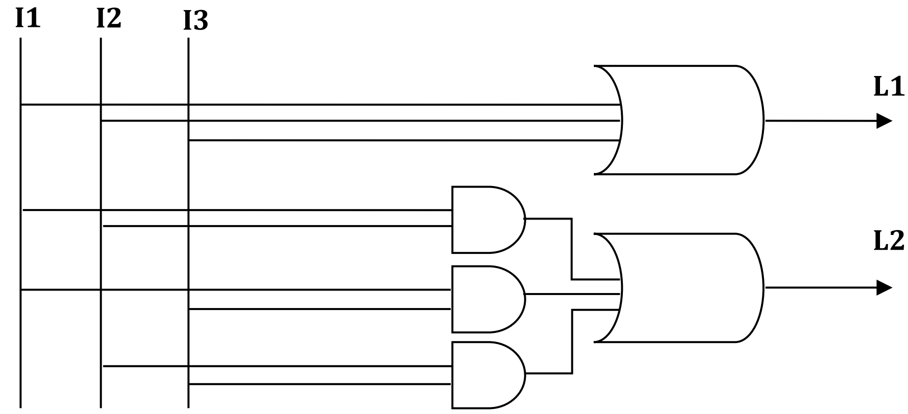
I2 I3 / I1	00	01	11	10
0	0	1	1	1
1	1	1	1	1

L2

I2 I3 / I1	00	01	11	10
0	0	0	1	0
1	0	1	1	1

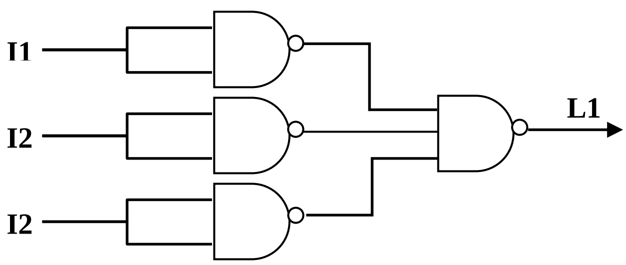
L1(I1, I2, I2) = I1 + I2 + I3 L2(I1, I2, I2) = I1.I2 + I1.I3 + I2.I3

2) **Flowcharts :**

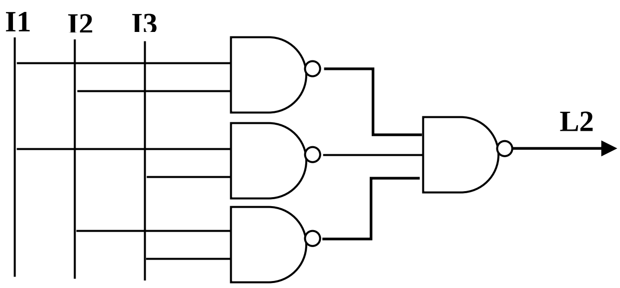


B. Expressions and representations with NAND

$$\begin{aligned} L1(I1, I2, I2) &= I1 + I2 + I3 \\ &= \overline{\overline{I1 + I2 + I3}} \\ &= \overline{\overline{I1} \,.\, \overline{I2} \,.\, \overline{I3}} \\ &= \overline{\overline{I1.I1} \,.\, \overline{I2.I2} \,.\, \overline{I3.I3}} \end{aligned}$$

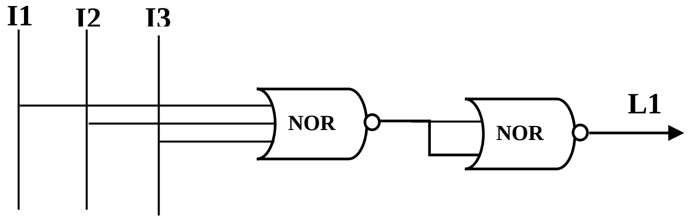


$$\begin{aligned} L2(I1, I2, I2) &= I1.I2 + I1.I3 + I2.I3 \\ &= \overline{\overline{I1.I2 + I1.I3 + I2.I3}} \\ &= \overline{\overline{I1.I2} \,.\, \overline{I1.I3} \,.\, \overline{I2.I3}} \end{aligned}$$



C. Expressions and representations with NOR

$$L1(I1, I2, I2) = I1 + I2 + I3$$
$$= \overline{\overline{I1 + I2 + I3}}$$
$$= \overline{\overline{I1 + I2 + I3} + \overline{I1 + I2 + I3}}$$



$$L2(I1, I2, I2) = I1.I2 + I1.I3 + I2.I3$$
$$= \overline{\overline{I1.I2 + I1.I3 + I2.I3}}$$
$$= \overline{\overline{\overline{\overline{I1.I2}} + \overline{\overline{I1.I3}} + \overline{\overline{I2.I3}}}}$$
$$= \overline{\overline{\overline{\overline{I1} + \overline{I2}} + \overline{\overline{I1} + \overline{I3}} + \overline{\overline{I2} + \overline{I3}}}}$$
$$= \overline{\overline{\overline{\overline{I1 + I1} + \overline{I2 + I2}} + \overline{\overline{I1 + I1} + \overline{I3 + I3}} + \overline{\overline{I2 + I2} + \overline{I3 + I3}}}}$$
$$= \overline{\overline{\overline{\overline{I1 + I1} + \overline{I2 + I2}} + \overline{\overline{I1 + I1} + \overline{I3 + I3}} + \overline{\overline{I2 + I2} + \overline{I3 + I3}}} + \overline{\overline{\overline{I1 + I1} + \overline{I2 + I2}} + \overline{\overline{I1 + I1} + \overline{I3 + I3}} + \overline{\overline{I2 + I2} + \overline{I3 + I3}}}}$$