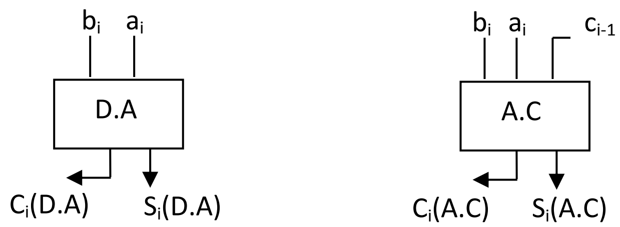
Solution to series 02 exercises

Exercise 1



$$S_i(H.A) = a_i \oplus b_i$$

$$C_i(H.A) = a_i b_i$$

$$S_i(F.A) = a_i \oplus b_i \oplus c_{i-1}$$

$$= S_i(H.A) \oplus c_{i-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(1)}$$

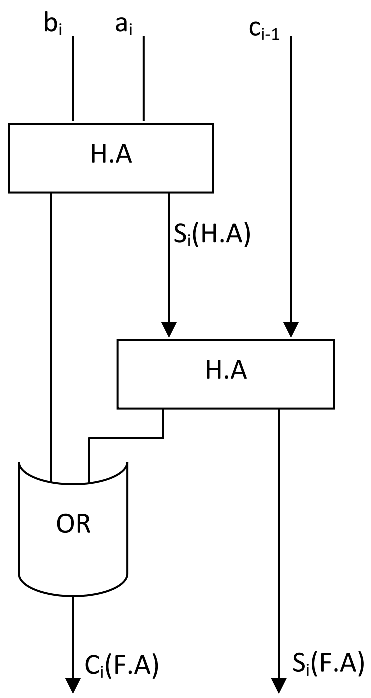
$$C_i(F.A) = \overline{c_{i-1}} a_i b_i + c_{i-1} \bar{a}_i b_i + c_{i-1} a_i \bar{b}_i + c_{i-1} a_i b_i$$

$$= c_{i-1}(a_i \bar{b}_i + \bar{a}_i b_i) + a_i b_i (\overline{c_{i-1}} + c_{i-1})$$

$$= c_{i-1}(a_i \oplus b_i) + a_i b_i (1)$$

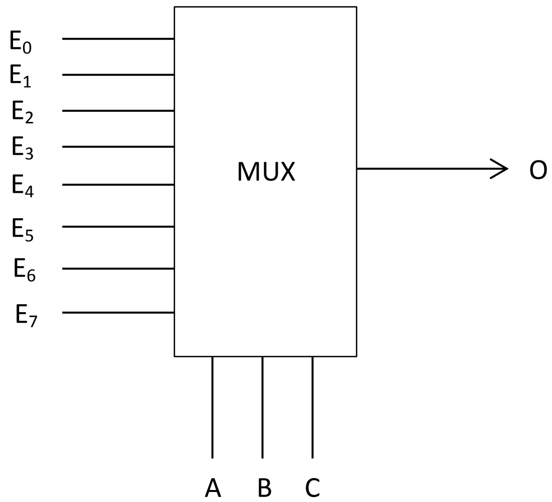
$$= c_{i-1}\, S_i(H.A) + a_i b_i$$

$$= c_{i-1}\, S_i(H.A) + C_i(H.A) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (\mathbf{2})$$



Exercise 2

1. We have the formula for the function:
   $F(A,B,C) = \bar{A}B\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}BCD + A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$
2. We will find the formula of the MUX with 3 selection lines
   The general notation of a multiplexer is as follows: MUX 2^n to 1,
   such that *n* represents the number of selection lines.
   We have 3 selection lines, so *n* = 3.
   Then the notation will be as follows: MUX 2^3 to 1 ≡ MUX 8 to 1
   Then the number of Entries (Inputs) is 8. The general diagram will be as follows:



The truth table is:

A	B	C	O
0	0	0	E_0
0	0	1	E_1
0	1	0	E2
0	1	1	E_3
1	0	0	E4
1	0	1	E5
1	1	0	E_6
1	1	1	E7

The formula for the MUX output will be as follows:

$$O = \bar{A}\bar{B}\bar{C}E_0 + \bar{A}\bar{B}CE_1 + \bar{A}B\bar{C}E_2 + \bar{A}BCE_3 + A\bar{B}\bar{C}E_4 + A\bar{B}CE_5 + AB\bar{C}E_6 + ABCE_7$$

3. We will now compare the two formulas O and F term by term to find the correspondences between the variables.
   The two formulas have the three variables A, B, and C in common. The difference lies in the 4th variable. For function F, the variable is D while for Output O, the 4th variable is one of E_i, i = 0, 1, …, 7.
   $F(A,B,C) = \bar{A}B\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}BCD + A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$

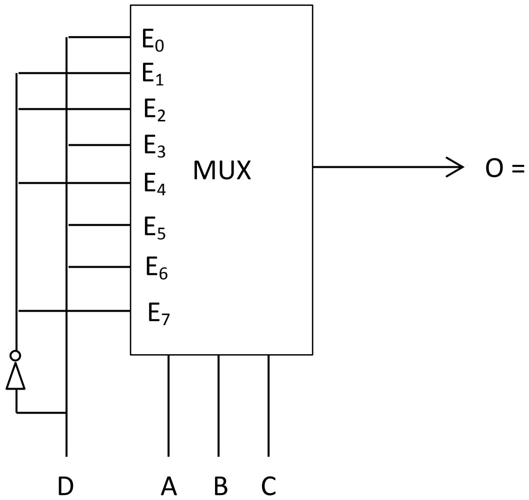
$$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$$

$$O = \bar{A}\bar{B}\bar{C}E_0 + \bar{A}\bar{B}CE_1 + \bar{A}B\bar{C}E_2 + \bar{A}BCE_3 + A\bar{B}\bar{C}E_4 + A\bar{B}CE_5 + AB\bar{C}E_6 + ABCE_7$$

So, by comparing the two formulas we find:

$$\left\{ E_0 = D, E_1 = \overline{D}, E_2 = \overline{D}, E_3 = D, E_4 = \overline{D}, E_5 = D, E_6 = D, E_7 = \overline{D} \right\}$$

From where, we represent the function F as follows



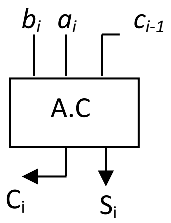
Exercise 3

1. The Full Adder and the Half Adder

1.1. The Full Adder

Truth table:

c_{i-1}	a_i	b_i	S_i	C_i
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Karnaugh table and Simplification:

C_i — $a_i b_i$ \ c_{i-1}	00	01	11	10
0	0	0	1	0
1	0	1	1	1

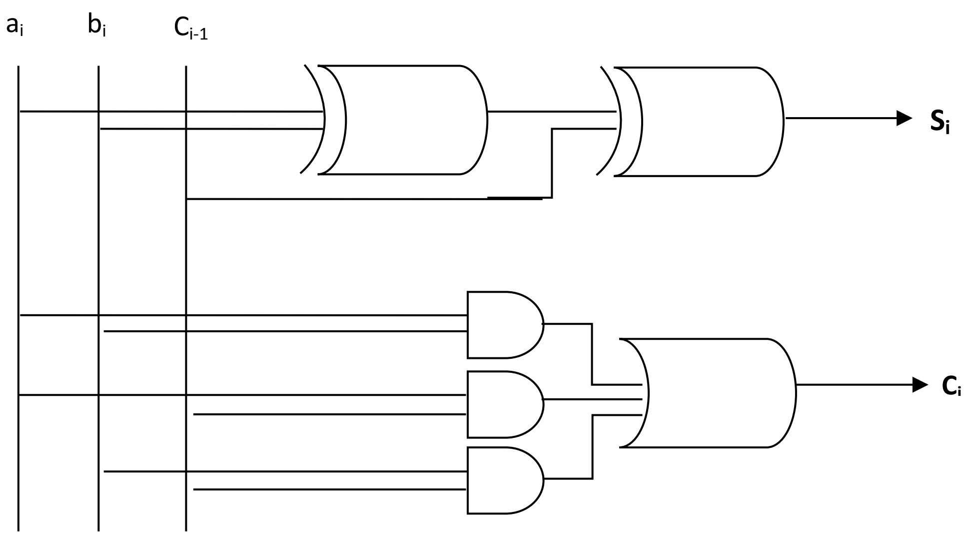
S_i — $a_i b_i$ \ c_{i-1}	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$C_i = a_i bi + c_{i-1} a_i + c_{i-1} b_i$$

$$S_i = \overline{c_{i-1}}\,\bar{a}_i b_i + \overline{c_{i-1}}\,a_i \bar{b}_i + c_{i-1}\bar{a}_i \bar{b}_i + c_{i-1} a_i b_i \quad (*)$$

$$= a_i \oplus b_i \oplus c_{i-1} \quad (**)$$

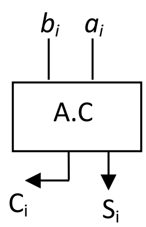
Flowchart: for S_i, the representation could be made by (*) or (**)



1.2. The Half Adder

Truth table:

a_i	b_i	S_i	C_i
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



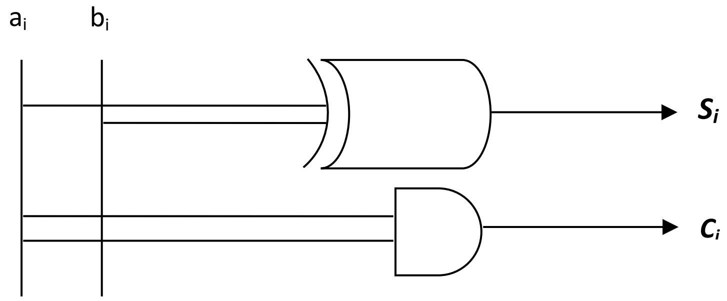
Simplification : directly from the truth table

$$S_i = a_i \bar{b}_i + \bar{a}_i b_i \quad (*)$$

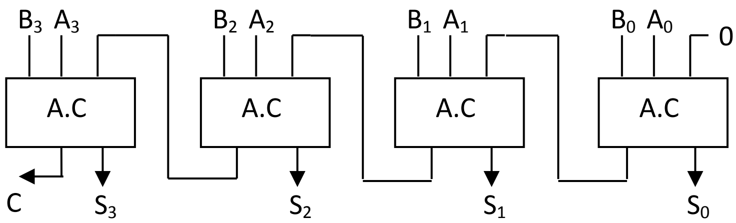
$$= a_i \oplus b_i \quad (**)$$

$C_i = a_i b_i$

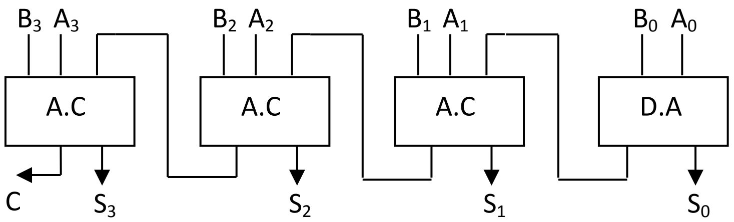
<u>Flowchart:</u> for S_i, the representation could be made by (*) or (**)



2. **<u>The adder of A and B using full adders only</u>**

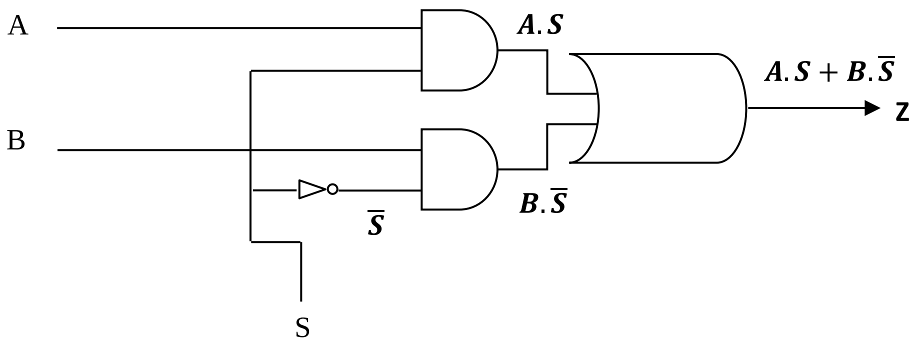


3. **<u>Adding A and B using a Half Adder and 3 Full Adders</u>**



<u>Exercise 4</u>

1. **<u>The expression of Z and the role of the circuit</u>**



$\Rightarrow \ Z = A.S + B.\overline{S}$

We will establish the truth table:

S	*HAS*	*B*	*Z*
0	0	**0**	**0**
0	0	**1**	**1**
0	1	**0**	**0**
0	1	**1**	**1**
1	**0**	0	**0**
1	**0**	1	**0**
1	**1**	0	**1**
1	**1**	1	**1**

Rows 1–4: The value of *B*

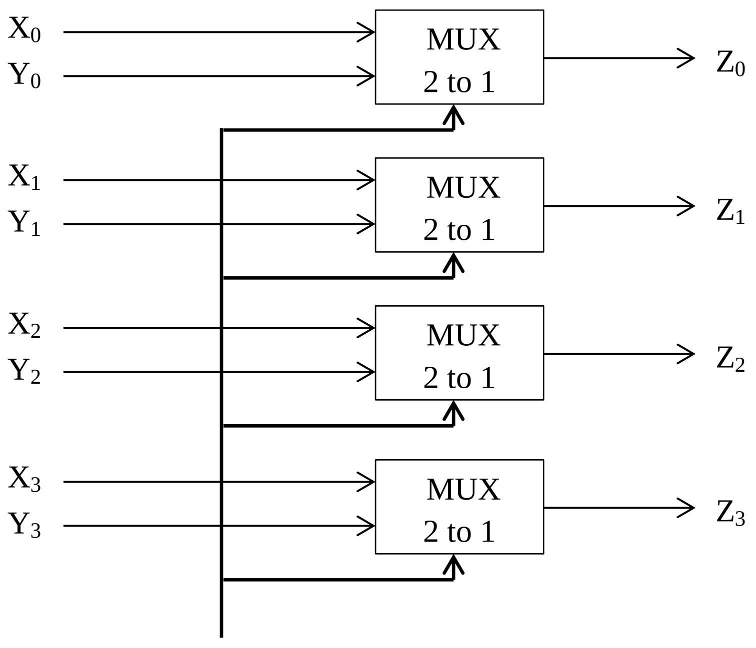
Rows 5–8: The value of *A*

⇒ **The circuit allows one bit to be selected at a time.**

⇒ **It is a multiplexer: MUX 2 to 1.**

2. <u>Direct creation of the circuit</u>

Let be the numbers: X ($X_3 X_2 X_1 X_0$) and Y($Y_3Y_2Y_1Y_0$) ⇒ to select the 4 bits of a number, we will use 4 multiplexers: MUX 2 to 1.

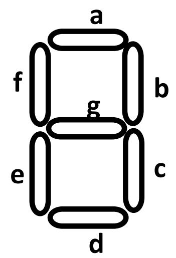


<u>Exercise 5</u>

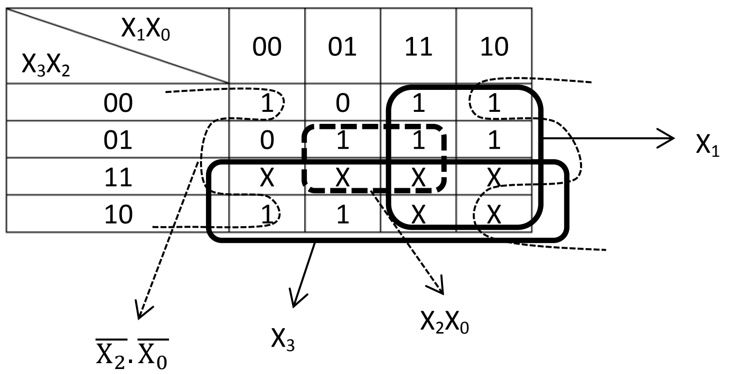
1. <u>The truth table:</u>

X_3	x_2	X_1	X_0	has	b	vs	d	e	f	g
0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**0**
0	0	0	1	**0**	**1**	**1**	**0**	**0**	**0**	**0**
0	0	1	0	**1**	**1**	**0**	**1**	**1**	**0**	**1**
0	0	1	1	**1**	**1**	**1**	**1**	**0**	**0**	**1**
0	1	0	0	**0**	**1**	**1**	**0**	**0**	**1**	**1**
0	1	0	1	**1**	**0**	**1**	**1**	**0**	**1**	**1**

0	1	1	0	**1**	**0**	**1**	**1**	**1**	**1**	**1**
0	1	1	1	**1**	**1**	**1**	**0**	**0**	**0**	**0**
1	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	0	0	1	**1**	**1**	**1**	**1**	**0**	**1**	**1**



2. Simplification of the function corresponding to segment (a)



3. Representation

$$a = X_1 + X_3 + X_2.X_0 + \overline{X_2}.\overline{X_0}$$