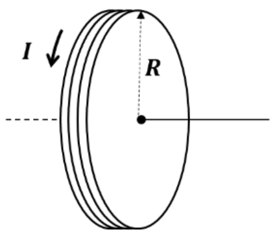
Series n°5: Electromagnetism

Exercise 1

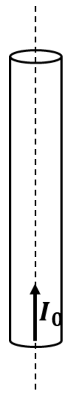
A flat circular coil of negligible thickness has $\boldsymbol{N}$ circular turns of radius $\boldsymbol{R}$ traversed by a current of intensity $\boldsymbol{I}$. (**Figure opposite**). Using the law of **Biot and Savart**:

1- Determine the magnetic field $\boldsymbol{B}$ created at the center of the coil.



Exercise 2

We consider a cylindrical conductor of radius $\boldsymbol{R}$ and infinite length $\boldsymbol{l}$ traversed by a current of intensity $\boldsymbol{I_0}$. The current density $\boldsymbol{J}$ is constant across the entire section of the cylinder and parallel to the axis Oz. Using **Ampère's theorem**:

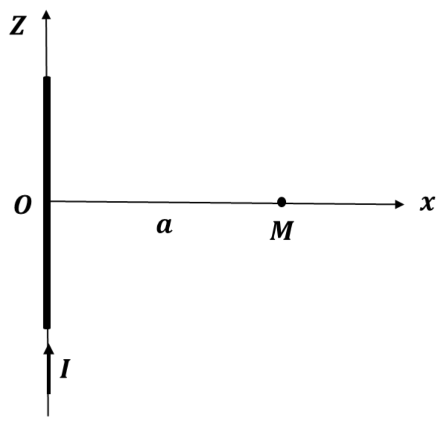


1- Determine the magnetic field $\boldsymbol{B}$ outside the cylinder $(r \geq R)$.
2- Determine the magnetic field $\boldsymbol{B}$ inside the cylinder $(r < R)$.
3- Draw the curve $\boldsymbol{B}(\boldsymbol{r})$.

Exercise 3 (Do in the course)

We consider a wire of infinite length and negligible diameter, traversed by an electric current of intensity $\boldsymbol{I}$ (**Figure below**). Using the law of **Biot and Savart**:

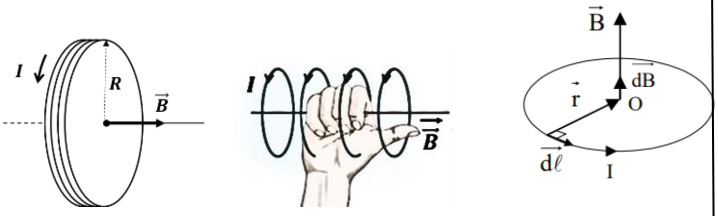
1- Determine the magnetic field $\vec{\boldsymbol{B}}$ created by this wire at a point $\boldsymbol{M}$ located on the $\boldsymbol{Ox}$ axis.



Exo- 1

1) Magnetic field created at the center of the coil

By applying the Right-Hand Thumb Rule (Maxwell's Corkscrew Rule), the magnetic field is carried along the axis of the circle and its direction depends on the direction of the current.



By applying the law of **Biot and Savart**:

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi\, r^3}\,\overrightarrow{dl} \wedge \vec{r}$$

$$\Rightarrow dB = \frac{\mu_0 I}{4\pi\, r^3}\,\|\overrightarrow{dl} \wedge \vec{r}\| = \frac{\mu_0 I}{4\pi\, r^3}\, r\, dl \sin\frac{\pi}{2};$$

$$and:\ \ r = R$$

$$\Rightarrow dB = \frac{\mu_0 I}{4\pi\, R^2}\, dl \qquad \Rightarrow B = \frac{\mu_0 I}{4\pi\, R^2}\, l$$

After integration over the length of the coil: $l = N2\pi R$

$$\Rightarrow \boldsymbol{B} = \frac{\mu_0\, \boldsymbol{N}\, \boldsymbol{I}}{\boldsymbol{2}\, \boldsymbol{R}}$$

Exo- 2

1) Magnetic field outside the cylinder $(r \geq R)$:

By applying **Ampère's theorem**: the circulation of the magnetic field in the closed trajectory (C) is:

$$C = \oint \vec{B}.\overrightarrow{dl} = \mu_0\, I_0$$

$$\Rightarrow\ Bl = \mu_0\, I_0 \qquad ;\ l\ =\ 2\pi\, r$$

$$\Rightarrow\ B2\pi\, r = \mu_0\, I_0$$

$$\Rightarrow\ \boldsymbol{B_1} = \frac{\mu_0\, \boldsymbol{I_0}}{\boldsymbol{2\pi\, r}}$$

2) Magnetic field inside the cylinder $(r\ <\ R)$:

As for the interior of the cylinder (r < R), the current passing through the section $\boldsymbol{S}$ is $\boldsymbol{I}$. The current density $\boldsymbol{J}$ is constant across the entire section of the cylinder, i.e:

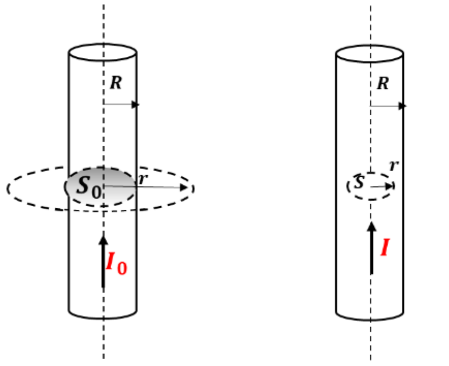
$$J = \frac{I_0}{S_0} = \frac{I}{S}$$

$$S_0 = \pi\, R^2;\ \ S = \pi\, r^2 \Rightarrow \boldsymbol{I} = \boldsymbol{I_0}\,\frac{\boldsymbol{r^2}}{\boldsymbol{R^2}}$$

The circulation is therefore equal to:

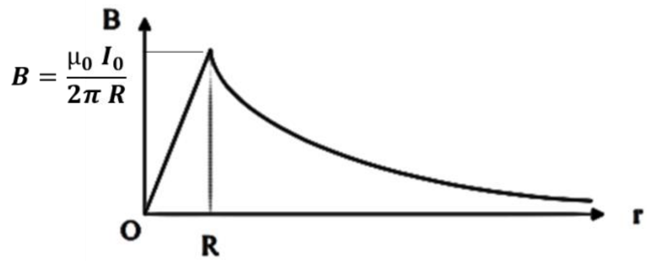
$$C = \oint \vec{B}.\overrightarrow{dl} = \mu_0 I \quad \Rightarrow\ Bl = \mu_0\, I$$

$$\Rightarrow\ \boldsymbol{B_2} = \frac{\mu_0\, \boldsymbol{I_0}}{\boldsymbol{2\pi\, R^2}}\boldsymbol{r}$$



$(r\ \geq R)$ $\qquad$ $(r\ <\ R)$

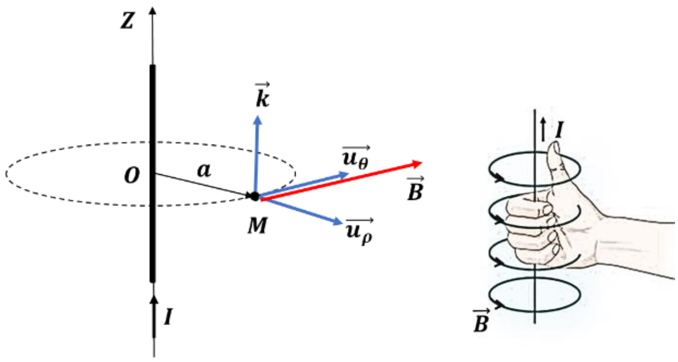
3) The curve $B(r)$.



Exo-3

The magnetic field $\vec{B}(M)$ created by the wire at a point $''M''$ in space, distant $''\boldsymbol{a}''$ from the wire, is defined in the cylindrical frame $(\overrightarrow{u_\rho}, \overrightarrow{u_\theta}, \vec{k})$ by:

$$\vec{B}(M) = B_\rho\, \overrightarrow{u_\rho} + B_\theta\, \overrightarrow{u_\theta} + B_z\, \vec{k}$$



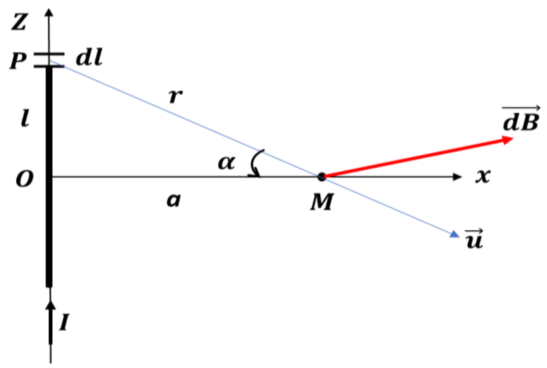
According to the corkscrew rule and the direction of the current $\boldsymbol{I}$, the magnetic field created by the wire at point M is tangential to the circle of radius a, this means that:

$$B_\rho = B_z = 0$$

$$\Rightarrow \vec{B}(M) = B_\theta\, \overrightarrow{u_\theta}$$

According to the law of **Biot and Savart**:

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi\, r^3}\,\overrightarrow{dl} \wedge \vec{r}$$



According to the diagram: $\begin{cases} r = PM \\ \vec{r} = \frac{\overrightarrow{PM}}{PM} \end{cases}$

We then write:

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi\, PM^3}\, \overrightarrow{dl} \wedge \overrightarrow{PM}$$

According to the diagram: $\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$

$$\Rightarrow \begin{cases} \overrightarrow{PM} = -l\,\vec{k} + a\,\overrightarrow{u_\rho} \\ \overrightarrow{dl} = dl\,\vec{k} \end{cases}$$

Let's calculate the vector product:

$$\overrightarrow{dl} \wedge \overrightarrow{PM} = \begin{vmatrix} \overrightarrow{u_\rho} & -\overrightarrow{u_\theta} & \vec{k} \\ 0 & 0 & dl \\ a & 0 & -l \end{vmatrix} = a\, dl\, \overrightarrow{u_\theta}$$

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi\, r^3}\, a\, dl\, \overrightarrow{u_\theta}$$

$$\boldsymbol{dB = \frac{\mu_0 I}{4\pi\, r^3}\, a\, dl} \quad \dots\dots (*)$$

As long as "l" is infinite, the angle α is between $[-\frac{\pi}{2}, \frac{\pi}{2}]$, we therefore change the variable from dl to $d\alpha$.

According to the diagram:

$$\begin{cases} \cos\alpha = \frac{a}{r} \\ \tan\alpha = \frac{l}{a} \end{cases} \Rightarrow \begin{cases} r = \frac{a}{\cos\alpha} \\ l = a\tan\alpha \Rightarrow dl = \frac{a}{cos^2\alpha}\, d\alpha \end{cases}$$

After replacing r and dl in $eq\ (*)$:

$$\Rightarrow dB = \frac{\mu_0 I}{4\pi\, a} \cos\alpha\, d\alpha$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi\, a} \int_{-\pi/2}^{\pi/2} \cos\alpha\, d\alpha$$

$$\boldsymbol{\vec{B}(M) = \frac{\mu_0 I}{2\pi\, a}\, \overrightarrow{u_\theta}}$$