Series n°4: Electrokinetics

Exercise 1

We consider a copper wire of section $\mathbf{S = 10\ mm^2}$ and length $\mathbf{l = 1m}$ traversed by a current of intensity $\mathbf{I = 1\ A}$ under a voltage $\mathbf{U = 20\ V}$, whose charge carriers are the electrons.

1- Calculate the number of electrons $\boldsymbol{n}$ per unit volume (assuming that each copper atom releases two electrons).
2- Calculate the current density $\boldsymbol{J}$ in the copper wire.
3- Deduce the drift velocity $\boldsymbol{V_D}$ of the electrons in the copper wire.
4- Calculate the conductivity $\boldsymbol{\sigma}$ of the copper wire.

We give:

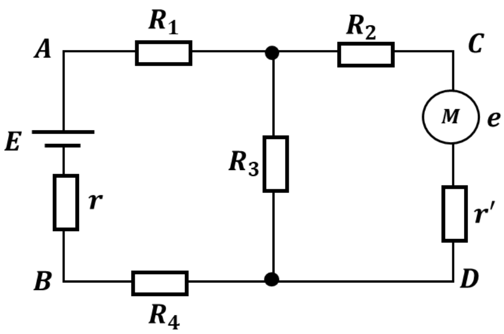
$$\boldsymbol{M_{Cu}} = 63.5g/mol\ ;\ \boldsymbol{N_A} = 6.02 \times 10^{23}\ atoms.mol^{-1}\ ;\ \boldsymbol{\rho} = 8800\ kg/m^3$$

Exercise 2

Consider the electrical circuit shown in the **figure opposite**. The current circulating in each branch is given by:

$I_1 = 1A;\ I_2 = 0.5\ A;\ \ and\ I_3 = 0.5\ A$.

1- Calculate the potential differences U_{AB} and U_{CD}.
2- Calculate the efficiency of the generator and the motor.
3- Calculate the power dissipated by the Joule effect in the circuit.



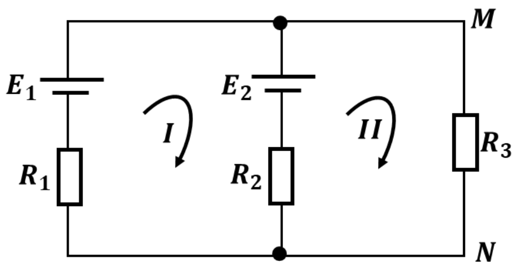
We give:

$$E = 10\ V,\ e = 2V,\ r = r' = 1\Omega,\ R_1 = 4\Omega,\ R_2 = 1\Omega,\ R_3 = 6\Omega\ and\ R_4 = 2\Omega$$

Exercise 3

Consider the electrical circuit illustrated in the **figure opposite**

1- Calculate current intensities I_1, I_2 and I_3 circulating respectively in the resistors R_1, R_2 and R_3 using **Kirchhoff's laws.**
2- Calculate the current I_3 circulating in the MN branch using **Thevenin's theorem**.



We give:

$$E_1 = 10\ V, E_2 = 8\ V, R_1 = R_2 = R_3 = 2\ \Omega.$$

N.B for exercise 2. You can recalculate the current in each branch, using **Kirchhoff's laws**.

Exo- 1

1) Calculating the number of electrons per unit volume.

- The number of Cu atoms contained in a volume of 1 m^3 is given by:

$$N = \frac{N_A .\ \rho}{M} = 8.34 \times 10^{28}\ atoms/m^3$$

- The number of electrons per unit volume is:

$$n = 2 \times N = 1.67 \times 10^{29}\ electrons/m^3$$

2) Calculating the current density J

$$we\ have:\ J = \frac{I}{S} = 10^5\ A/m^2$$

2) Calculating the drift velocity V_D

$$we\ have:\ J = n\ e\ V_D \Rightarrow V_D = \frac{J}{n\ e} = 3.74 \times 10^{-6}\ m/s$$

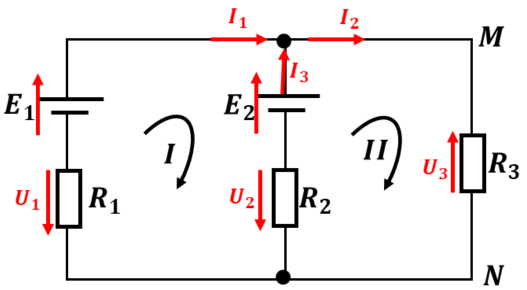
3) Calculating the conductivity σ

$$\text{we have}:\ \sigma = \frac{l}{R.S} = \frac{I.l}{U.S} = 5 \times 10^3\ (\Omega.m)^{-1}$$

Exo- 3

1) Calculating the current I_1, I_2 and I_3

Using **Kirchhoff's laws**.



- 1st law of Kirchhoff: current law (or law of the junctions):

$$I_1 + I_2 = I_3$$

- 2nd law of Kirchhoff: voltage law (or law of the loops):

Loop I : $E_1 - E_2 + U_2 - U_1 = 0$

$$E_1 - E_2 + R_2 I_2 - R_1 I_1 = 0$$
$$10 - 8 + 2I_2 - 2I_1 = 0$$
$$I_1 - I_2 = 1$$

Loop II : $E_2 - U_3 - U_2 = 0$

$$E_2 - R_3 I_3 - R_2 I_2 = 0$$
$$8 - 2I_3 - 2I_2 = 0$$
$$I_2 + I_3 = 4$$

We obtain a system of 3-equations with 03-unknowns:

$$\begin{cases} I_1 + I_2 - I_3 = 0 \dots \dots . \dots . (1) \\ I_1 - I_2 = 1 \dots \dots \dots \dots \dots \dots (2) \\ I_2 + I_3 = 4 \dots \dots \dots \dots \dots \dots (3) \end{cases}$$

To eliminate I_3 we do $eq(1) + eq(3)$, we arrive at the system:

$$\begin{cases} I_1 - I_2 = 1 \dots \dots \dots \dots \dots \dots (2) \\ I_1 + 2I_2 = 4 \dots \dots \dots \dots \dots . (4) \end{cases}$$

To eliminate I_1 we do $eq(4) - eq(2)$, we arrive at:

$$I_2 = 1\ A$$

we remplace I_2 in $eqs\ (2)\ and\ (3)$:

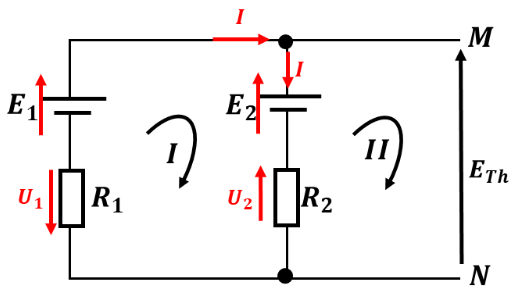
$$I_1 = 2\ A$$
$$I_3 = 3\ A$$

For verification: we replace I_1, I_2 and I_3 in $eq\ (1)$

2) Calculating the current I_3

By applying Thevenin's theorem

a) Calculating E_{th} (e.m.f of generator of Thevenin)



$$E_{th} = U_{MN} = ?$$

Loop I : $E_1 - E_2 - U_2 - U_1 = 0$

$$E_1 - E_2 - R_2 I - R_1 I = 0$$
$$I = \frac{E_1 - E_2}{R_1 + R_2}$$
$$I = 0.5\ A$$

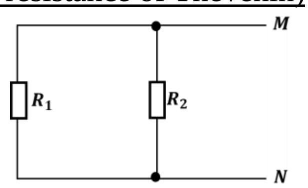
Loop II : $E_2 - E_{th} + U_2 = 0$

$$E_2 - E_{th} + R_2 I = 0$$
$$E_{th} = E_2 + R_2 I$$
$$E_{th} = 9\ V$$

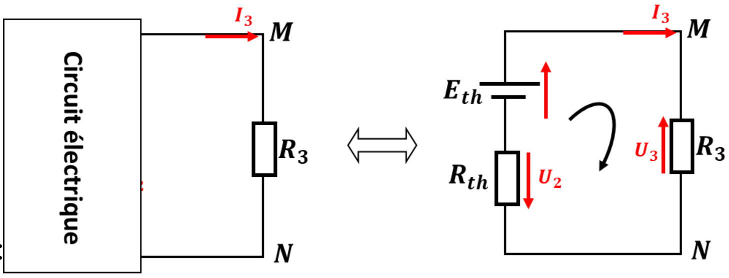
b) Calculating R_{th} (equivalent resistance of Thevenin)

$$\frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{th} = 1\ \Omega$$



c) Calculating I_3 (circulating in the MN branch)



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Loop: $E_{th} - R_3 I_3 - R_{th} I_3 = 0$

$$I_3 = \frac{E_{th}}{R_3 + R_{th}} \Rightarrow \boldsymbol{I_3 = 3\ A}$$

Exo-2

1) Potential differences U_{AB} and U_{CD}.

$U_{AB} = E - rI_1 = 9\ V$

$U_{CD} = e + rI_2 = 2.5\ V$

2) The efficiency of the generator and the motor.

The efficiency of a generator is the ratio of the useful power to the total power supplied by the generator.

The efficiency of a motor is the ratio of useful power to absorbed power. The absorbed power represents the electrical power that enters the motor. Useful power corresponds to the electrical power that comes out of the motor.

$$\boldsymbol{r_G} = \frac{P_u}{p_G} = \frac{P_{AB}}{p_G} = \frac{U_{AB}}{E} = \mathbf{90}\ \%$$

$$\boldsymbol{r_M} = \frac{P_u}{p_a} = \frac{P_M}{p_{CD}} = \frac{e}{U_{CD}} = \mathbf{80}\ \%$$

3) The power dissipated by the joule effect in the circuit.

$P = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_1^2 + rI_1^2 + r' I_2^2$

$\boldsymbol{P = 9\ W}$