1 Multidimensional linear problem with constraints

A multidimensional linear problem, also known as a linear programming problem, involves optimizing a linear objective function subject to linear constraints. These problems are prevalent in various fields such as operations research, economics, engineering, and finance.

Here's how you can implement this in MATLAB:

1.1 Command "linprog"

To solve a multidimensional linear problem with constraints in MATLAB, you can use the built-in function linprog. This function is designed to solve linear programming problems of the form:

Minimize:

$f(x) = c^T x$

Subject to:

$Ax \leq b$

$lb \leq x \leq ub$

Here's how you can use linprog for your problem:

```
% Define the objective function coefficients
c = [1; 2; 3]; % Coefficients of the objective function
% Define the inequality constraints: A*x <= b
A = [1, 1, 1; % Coefficients of the first constraint
 2, 3, 1]; % Coefficients of the second constraint
b = [10; 20]; % Right-hand side of the constraints
% Define the bounds for variables: lb <= x <= ub
lb = zeros(3,1); % Lower bounds for variables
ub = []; % Upper bounds for variables (empty for unbounded)
```

```
% Solve the linear programming problem
[x, fval, exitflag] = linprog(c, [], [], A, b, lb, ub);
% Display the results
if exitflag == 1
 disp('Optimal solution found:');
 disp(['x = ', num2str(x')]);
 disp(['Optimal objective value = ', num2str(fval)]);
else
 disp('Optimal solution not found.');
end
```

Example 1 *Let consider the function* $f(x) = x_1 + 2x_2 + 3x_3$ *subject to the following constraints:*

$x_1 + x_2 + x_3 \leq 15$
$2x_1 + 3x_2 + x_3 \leq 25$
$x_1, x_2, x_3 \geq 0$

Here's how you can solve this problem using linprog in MATLAB:

```
c = [1; 2; 3];
A = [1, 1, 1;
 2, 3, 1];
b = [15; 25];
lb = zeros(3,1);
ub = [];
[x, fval, exitflag] = linprog(c, [], [], A, b, lb, ub);
if exitflag == 1
 disp('Optimal solution found:');
 disp(['x = ', num2str(x')]);
 disp(['Optimal objective value = ', num2str(fval)]);
else
 disp('Optimal solution not found.');
end
```

2 Multidimensional non-linear problem with constraints

2.1 Command "fmincon"

To solve a multidimensional non-linear problem with constraints in MATLAB, you can use the fmincon function. fmincon is a versatile optimization function capable of handling nonlinear constraints. Here's a basic example demonstrating how to use fmincon:

$f(x) = (x1 - 2)2 + (x2 - 3)2$

Subject to the following constraints:

$x_1 + x_2 \leq 5$

$x_1 - x_2 \geq 2$
$x_1, x_2 \geq 0$
Here's how you can implement this in MATLAB using fmincon:

```
% Define the objective function
fun = @(x) (x(1) - 2)^2 + (x(2) - 3)^2;
% Initial guess for x
x0 = [0; 0];
% Define the nonlinear inequality constraints
nonlcon = @(x) constraints(x);
% Solve the optimization problem
[x, fval, exitflag, output] = fmincon(fun, x0, [], [], [], [], [], [], nonlcon);
% Display the results
if exitflag > 0
 disp('Optimal solution found:');
 disp(['x = ', num2str(x')]);
 disp(['Optimal objective value = ', num2str(fval)]);
else
 disp('Optimal solution not found.');
end
% Define the nonlinear inequality constraints
function [c, ceq] = constraints(x)
 c = [x(1) + x(2) - 5; % x1 + x2 <= 5
 -(x(1) - x(2) - 2)]; % x1 - x2 >= 2
 ceq = []; % No equality constraints
end
```

Suppose we want to minimize the following function:
$f(x)$
Subject to the following constraints:
$g(x) = 4$
$h(x) \geq 1$
$x = (x_1, x_2)\, with\ \ x_1, x_2 \geq 0$
Here's how you can implement this in MATLAB using fmincon:

```
fun = @(x) f(x);
x0 = [0; 0];
nonlcon = @(x) constraints(x);
Aeq = [1, 1];
beq = 4;
[x, fval, exitflag, output] = fmincon(fun, x0, [], [], Aeq, beq, [], [], nonlcon);
if exitflag > 0
 disp('Optimal solution found:');
 disp(['x = ', num2str(x')]);
 disp(['Optimal objective value = ', num2str(fval)]);
else
 disp('Optimal solution not found.');
end
```

```
function [c, ceq] = constraints(x)
 c = -(g(x) - 1); % g(x) >= 1
 ceq = []; % No equality constraints
end
```

———

Find the minimum value of Rosenbrock's function when there is a linear inequality constraint.

Find the minimum value starting from the point $[-1, 2]$, constrained to have $x(1) + 2x(2) \leq 1$.

Express this constraint in the form $Ax <= b$ by taking $A = [1, 2]$ and $b = 1$.

Here's how you can implement this in MATLAB using fmincon:

```
fun = @(x)100*(x(2)-x(1)^2)^2 + (1-x(1))^2;
x0 = [-1,2];
A = [1,2];
b = 1;
x = fmincon(fun,x0,A,b)
```

———————————————

Find the minimum value of Rosenbrock's function when there are both a linear inequality constraint and a linear equality constraint.

Set the objective function fun to be Rosenbrock's function.

Find the minimum value starting from the point $[0.5, 0]$, constrained to have $x(1) + 2x(2) \leq 1$ and $2x(1) + x(2) = 1$.

Here's how you can implement this in MATLAB using fmincon:

```
fun = @(x)100*(x(2)-x(1)^2)^2 + (1-x(1))^2;
x0 = [0.5,0];
A = [1,2];
b = 1;
Aeq = [2,1];
beq = 1;
x = fmincon(fun,x0,A,b,Aeq,beq)
```