Chapter 2: The conductors in equilibrium النواقل المتزنة

1. Definitions

- An electrical conductor is a body in which the electric charges can freely move when the voltage is applied.
- We say that a conductor is in electrostatic equilibrium if it charges are in a state of rest.

2. Properties of a conductor in equilibrium خصائص النواقل المتزنة

1) The electrostatic field inside the conductor in equilibrium is null ($\overrightarrow{\boldsymbol{E_{int}}} = \vec{\mathbf{0}}$)

As long as the electric charges inside the conductor in equilibrium are at rest, they are therefore not subject to any force:

$$\vec{F_e} = \vec{0}$$

$$\vec{F_e} = q\vec{E} = \vec{0} \quad \Rightarrow \quad \vec{E} = \vec{0}$$

2) The conductor in equilibrium constitutes an equipotential volume ($\boldsymbol{i.e.}\ \boldsymbol{V_{int}} = \boldsymbol{C^t}$).

We have: $dV = -\vec{E}.\overrightarrow{dl} = 0 \Rightarrow V = C^t$

This implies that the potential is constant at each point of the conductor in equilibrium (inside and at the surface).

3) The charge inside the conductor in equilibrium is null ($\boldsymbol{Q_{int}} = \mathbf{0}$).

Because: $Q^- + Q^+ = 0$

If the conductor is charged the added charges are located on the surface.

4) If the conductor is charged, the field vector $\vec{E}$ is perpendicular to the external surface of the conductor in equilibrium.

As long as $V = C^t$*,the external surface of the conductor is an equipotential surface, which proves that the field is perpendicular to the surface of the conductor.*

$$dV = -\vec{E}.\overrightarrow{dl} = 0 \quad \Rightarrow \vec{E} \perp \overrightarrow{dl}$$

We can summarize the properties of the conductor in equilibrium in the following diagram:

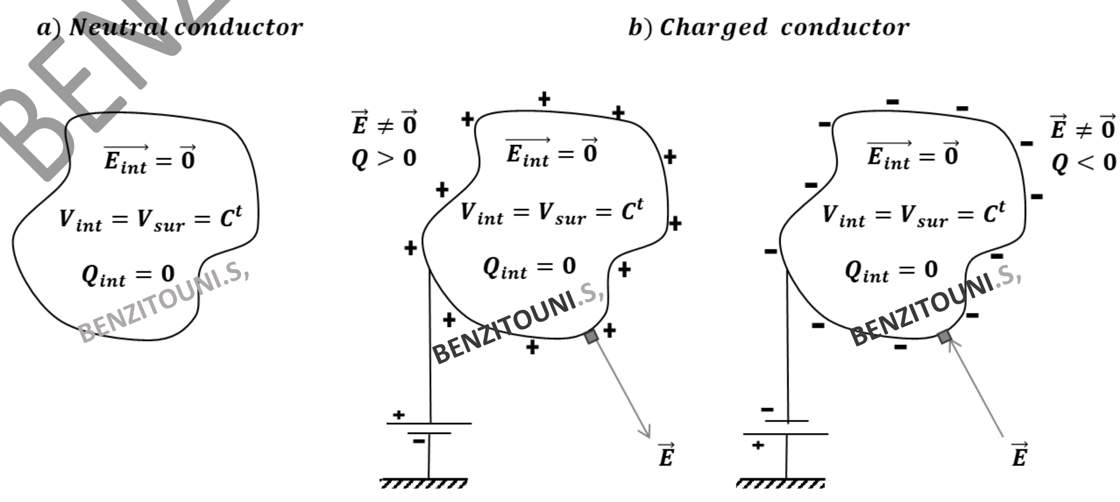


Figure.1. Summary of the properties of a conductor in equilibrium (neutral and charged).

3. Coulomb's theorem: The electric field near a conductor حقل كهربائي بالقرب من ناقل

In the immediate vicinity of a charged conductor in equilibrium (C) placed in a vacuum, the electric field is perpendicular to the surface of the conductor and its intensity E is equal to:

$$\boldsymbol{E = \frac{\sigma}{2\,\varepsilon_0}}$$

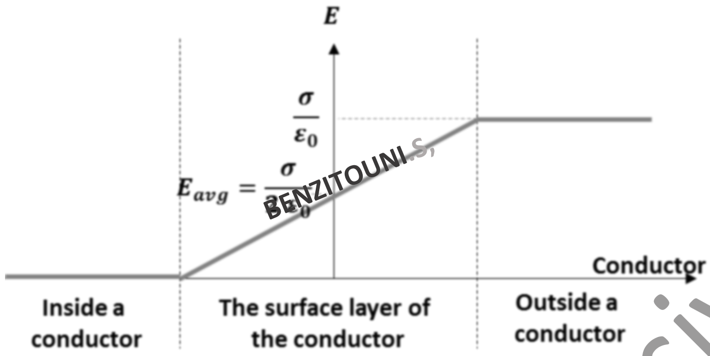


Figure.2. Variation of the electric field across the surface of the conductor.

Demonstration of Coulomb's theorem

We apply Gauss's Theorem:

$$\emptyset = \oiint \vec{E}.\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}$$

S_G: The Gaussian surface which is suitable here is a surface of a cylinder (r, h); where: S_G: (S_1, S_2, S_L)

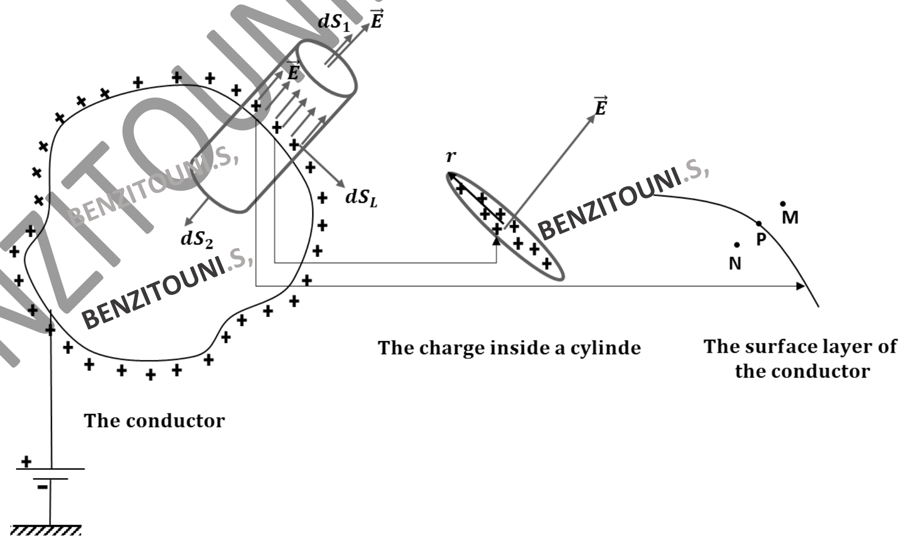


Figure.3. Demonstration of Coulomb's theorem

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

Where:

$$\begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = \boldsymbol{ES}; & (\vec{E} \parallel \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \mathbf{0}; & (\overrightarrow{E_{int}} = \vec{0}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_{L,int}} + \iint \vec{E}.\overrightarrow{dS_{L,ext}} = \mathbf{0}; & (\vec{E} \perp \overrightarrow{dS_{L,ext}}) \; and \; (\overrightarrow{E_{int}} = \vec{0}) \end{cases}$$

Therefore:

$$\emptyset_T = E.S = \frac{Q_{int}}{\varepsilon_0}$$

Knowing that: $\begin{cases} S = \pi r^2 \\ Q_{int} = \pi r^2 \sigma \end{cases}$

The electric field near a conductor is given by:

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\ \vec{n}$$

$\vec{\boldsymbol{n}}$: Normal unit vector.

This expression gives the electric field in the vicinity of the surface and outside the conductor (the point M), while the field inside is null (the point N). On the surface (In the immediate vicinity, the point P) the field takes an average value E_{avg} which can be presented as follows:

$$E_{avg} = \frac{E_N + E_M}{2}$$

$$\boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\mathbf{2}\,\boldsymbol{\varepsilon_0}}$$

4. Electrostatic pressure ضغط كهروستاتيكي

The charges located on the surface of the conductor are subjected to repulsive forces with other charges, which leads to pressure on the surface of the conductor and thus:

$$p = \frac{F_e}{S} = \frac{QE}{S} = \sigma\, E$$

$$\boldsymbol{p} = \frac{\boldsymbol{\sigma^2}}{\mathbf{2}\,\boldsymbol{\varepsilon_0}}$$

The unit of electrostatic pressure is the **Pascal** $(\boldsymbol{Pa})$**.**

5. The power of pointed surfaces قدرة الاسطح الحادة

Experimentally, the electric charges have a tendency to accumulate on pointed surfaces with small radius of curvature. The surface charge density is then important in the pointed zone, the same reasoning for the electric field in the vicinity of these pointed surfaces.

If the surface density σ is very high, the charges leave the conductor, this is known as field effect emission.

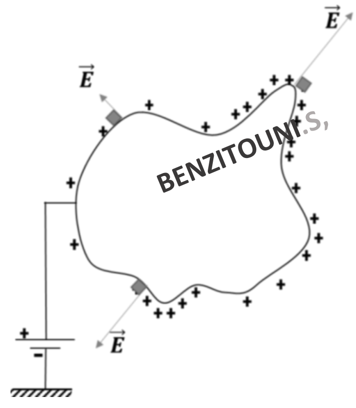


Figure.4. Pointed surfaces

Demonstration

Consider two spherical conductors (S_1, S_2), each with its own characteristics, connected by a wire.

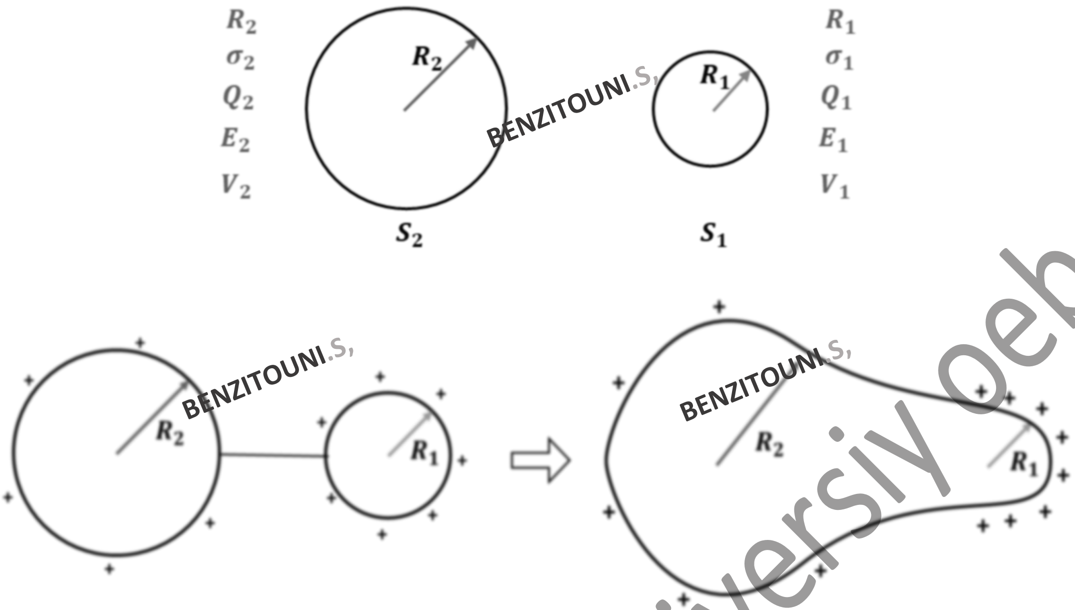


Figure.5. System of two connected spheres.

The whole constituting a single conductor with a new state of equilibrium, where: $V = C^t$

$$V_1 = V_2$$

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

$$\Rightarrow \frac{k\sigma_1 S_1}{R_1} = \frac{k\sigma_2 S_2}{R_2}$$

$$\Rightarrow \sigma_1 R_1 = \sigma_2 R_2$$

$$\Rightarrow \boldsymbol{\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}}$$

We have: $\boldsymbol{R_2 > R_1} \quad \Rightarrow \quad \boldsymbol{\sigma_2 < \sigma_1}$

This proves that charges tend to accumulate on pointed surfaces.

In life, we find the applications of pointed surfaces in the electrical discharge on the wings of planes, lightning arresters (antifoudre), lightning rods (paratonnerres), etc.

6. Electric Capacitance of a conductor السعة الكهربائية للناقل

The electric capacitance "C" of a conductor is the ratio between its charge "Q" and its potential "V":

$$C = \frac{Q}{V}$$

For the case of a spherical conductor placed in a vacuum, the capacitance is given by:

$$C = \frac{Q}{V} \quad and \quad V = \frac{kQ}{R}$$

Therefore: $$C = 4\,\pi\,\varepsilon_0\,R$$

If the insulating medium surrounding the spherical conductor is other than vacuum, its capacitance is then given by:

$$C = 4\,\pi\,\varepsilon\,R$$

ε : Dielectric permittivity of the medium.

For the case of two conductors carrying two charges $+Q$ and $-Q$, the capacitance of the system is given by:

$$C = \frac{Q}{U};\ U = V_1 - V_2$$

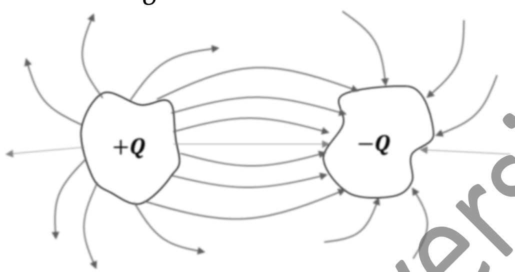


Figure.6. capacitance of two conductors

The unit of capacitance C is expressed in Farad (F).

$$[C] = \frac{[Q]}{[V]} = C/V = F$$

- $1\ mF = 10^{-3}\ F$
- $1\ \mu F = 10^{-6}\ F$
- $1\ nF = 10^{-9}\ F$
- $1\ pF = 10^{-12}\ F$

Example-1 :

Calculate the capacitance C of the earth. We give: $R = 6400\ km;\quad \varepsilon_0 = 8.85 \times 10^{-12}\ (SI)$

Sol : $C = 710\ \mu\ F$

Example-2 :

Calculate the radius of a spherical conductor with capacitance $C = 1\ \mu F$, placed in a vacuum

Sol : $R = 9009\ km.$

7. Electrostatic Potential energy of a charged conductor. طاقة كامنة للناقل مشحون

The electrostatic potential energy E_p stored in a conductor presents the work necessary to charge this conductor:

$$dE_p = dW = d(Vq) = Vdq = \frac{q}{C}dq$$

$$dE_p = \frac{1}{C}\int_0^Q q\,dq$$

$$E_p = \frac{1}{2}\frac{Q^2}{C};\ E_p = \frac{1}{2}CV^2;\ E_p = \frac{1}{2}QV$$

8. Influence phenomena between conductors ظواهر التأثير بين نواقل

What happens when we place a neutral and equilibrium conductor in an external electric field? : ***Polarization****.*

Why: When we put an equilibrium conductor in an external electric field $\overrightarrow{E_{ext}}$, the free charges (at rest) move: the positive charges move in the direction of $\overrightarrow{E_{ext}}$ while the negative charges move in the opposite direction. It then occurs **a polarization of the conductor**.

8.1. Partial influence تأثير جزئي

We place the charge $(+q)$ in the vicinity of the neutral conductor (**figure** below). This means that all field lines starting from the punctual charge $(+q)$ do not reach the entire conductor, and this is what characterizes the partial influence.

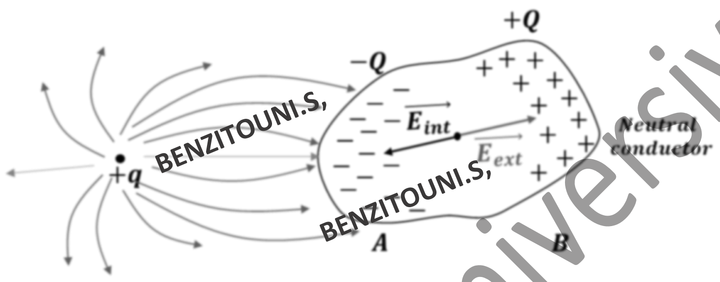


Figure.7. Partial influence

The charge (+q) produces an external field $\overrightarrow{E_{ext}}$, which forces the free electrons of the conductor to move towards face "A", this region becomes negatively charged. The electrons leaving the face "B" create a region of positive charge.

Then, the charges $+Q$ and $-Q$ in turn, produce an internal field $\overrightarrow{E_{\imath nt}}$ which opposes the direction of the external field $\overrightarrow{E_{ext}}$.

When $\overrightarrow{E_{ext}} = \overrightarrow{E_{\imath nt}}$, the movement of electrons stops and the conductor becomes in electrostatic equilibrium, despite its polarization. Where:

$$\text{Inside a conductor:} \begin{cases} \overrightarrow{E_{ext}} + \overrightarrow{E_{\imath nt}} = \vec{0} \\ V = C^t \\ Q_T = 0 \end{cases}$$

Notes:

- If the conductor is connected to the earth (where the potential is zero: V=0), the positive charges (+) move towards earth and the potential of the conductor is canceled (zero).
- If we remove the connection between the conductor and the earth and separated it from charge q, the conductor becomes permanently charged (by influence).

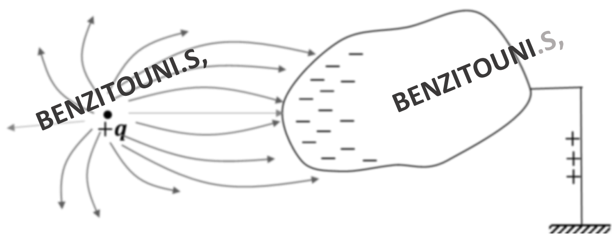


Figure.8. Polarized conductor connected to the earth.

8.2. Total influence التأثير الكلي

When a conductor (A) completely surrounds another conductor (B), all the lines of the field which are from (B) arrive at the conductor (A). This is what characterizes the total influence.

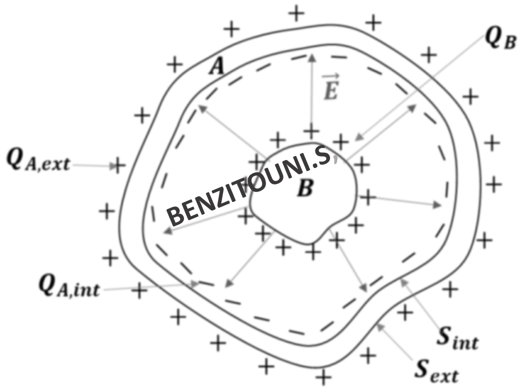


Figure.9. Total influence

a) If A is initially isolated and neutral

Principle of conservation of charge: $Q_{A,int} + Q_{A,ext} = 0$

According to the Theorem of corresponding elements نظرية العناصر المتناسبة:

$$\begin{cases} Q_B = -Q_{A,int} \\ Q_{A,int} = -\,Q_{A,ext} \end{cases} \Rightarrow \boldsymbol{Q_B = Q_{A,ext}}$$

b) If A is initially isolated and carries a charge Q_0

Principle of conservation of charge remains valid:

$$Q_0 = Q_{A,int} + Q_{A,ext}$$

According to the Theorem of corresponding elements:

$$Q_B = -Q_{A,int}$$

$$\Rightarrow \boldsymbol{Q_{A,ext} = Q_B + Q_0}$$

9. Electric capacitors مكثفات

The capacitor is an electronic component (device), made up of two conductors in total influence and separated by a dielectric medium (insulator). Its main property is to store and condense electrical energy by accumulating electric charges on its surfaces of its conductors. The capacitor was originally known as the condenser.

9.1. Planar Capacitor (or Parallel Plate Capacitor) مكثفة مستوية

A planar capacitor consists of two parallel metal plates, separated by a distance "d" by a dielectric medium of permittivity ε:

$$\boldsymbol{\varepsilon = \varepsilon_0\ \varepsilon_r}$$

The dielectric permittivity of the medium ε is the physical property which describes the response of the medium to the applied electric field.

$\boldsymbol{\varepsilon_0}$: The permittivity of a vacuum or free space, $\boldsymbol{\varepsilon_0 = 8.85\ \times 10^{-12} F/m}$.

ε_r: Relative permittivity of the insulator, also known as dielectric constant of an insulator

Table: Examples for relative permittivity of the same materials.

Insulator/material	Vacuum	Air	Glass	Polystyrene	Carbon disulfide
Relative permittivity	$\varepsilon_r = 1$	$\varepsilon_r = 1{,}0006$	$\varepsilon_r = 4.7$	$\varepsilon_r = 2.4 - 2.7$	$\varepsilon_r = 2.6$

1) The electrostatic field $\vec{E}$ between the plates

The field in the immediate vicinity of a conductor in equilibrium is given by:

$$\boldsymbol{E = \frac{\sigma}{2\varepsilon}\,;\ \varepsilon = \varepsilon_0\,\varepsilon_r}$$

As long as the capacitor is made up of two conductors (plates), each plates produces a field $\overrightarrow{E_i}$:

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}\,;\ \Rightarrow \vec{E} = 2\,E\,\vec{i} \Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0\,\varepsilon_r}\,\vec{i}$$

$$\Rightarrow \boldsymbol{\vec{E} = \frac{Q}{S\,\varepsilon_0\,\varepsilon_r}\,\vec{i}}\ ;\text{or } \boldsymbol{E = \frac{Q}{S\,\varepsilon_0\,\varepsilon_r}}$$

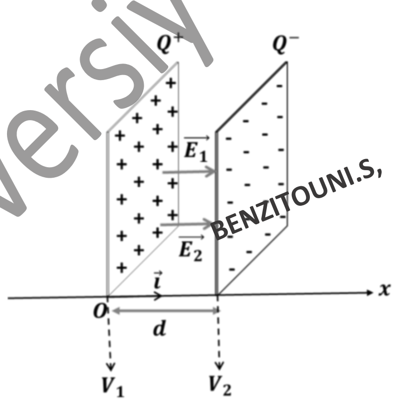


Figure.10. Planar capacitor

2) The potential difference U between the plates

We have : $U = V_1 - V_2$

Where $dV = -\vec{E}.\overrightarrow{dl} = -E\,dl \Rightarrow \int_{V_1}^{V_2} dV = -\int_0^d E\,dl$

$$\Rightarrow \boldsymbol{U = Ed = \frac{Q\,d}{S\,\varepsilon_0\,\varepsilon_r}}$$

3) The capacitance C

$$\text{we have : } C = \frac{Q}{U} = \boldsymbol{\frac{S\,\varepsilon_0\,\varepsilon_r}{d}}$$

4) Energy E_p stored in the capacitor

$$\text{we have: } E_p = \frac{1}{2}Q\,U \Rightarrow \boldsymbol{E_p = \frac{1}{2}\varepsilon_0\,\varepsilon_r\,E^2\,V}$$

where: $V = d.S$ (volume)

5) The volume density of energy can be given by**:**

$$\boldsymbol{\frac{dE_p}{dV} = \frac{1}{2}\varepsilon_0\,\varepsilon_r\,E^2}$$

9.2. Spherical capacitor مكثفة كروية

The spherical capacitor is made up of two concentric spherical conductors, separated by an insulator medium of permittivity ε.

1) The electrostatic field $\vec{E}$ between the spheres

T of Gauss :

$$\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0\boldsymbol{\varepsilon_r}}$$

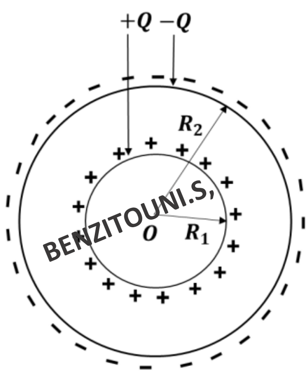


Figure.11. Spherical capacitor

S_G : Gaussian sphere with radius r : $S_G = 4\pi\, r^2$

Region-1: $r < R_1 : Q_{int} = 0 \Rightarrow E_1 = 0$

Region -2: $r > R_2 : Q_{int} = +Q + Q = 0 \Rightarrow E_2 = 0$

Region -3: $R_1 < r < R_2 : Q_{int} = +Q \Rightarrow E.4\,\pi\, r^2 = \frac{+Q}{\varepsilon_0 \varepsilon_r}$

$$\Rightarrow \boldsymbol{E = \frac{Q}{4\,\pi\varepsilon_0\, \varepsilon_r\, r^2}}$$

2) The potential difference U between the spheres

We have: $U = V_1 - V_2$ et $\vec{E} = -\overrightarrow{grad}\, V \quad \Rightarrow dV = -E\, dr$

$$\Rightarrow \boldsymbol{U = \frac{Q}{4\pi\, \varepsilon_0\, \varepsilon_r}\left(\frac{R_2 - R_1}{R_1 R_2}\right)}$$

3) Capacitance C.

we have : $C = \frac{Q}{U}$

$$\Rightarrow \boldsymbol{C = 4\pi\, \varepsilon_0\, \varepsilon_r \left(\frac{R_1 R_2}{R_2 - R_1}\right)}$$

4) Energy E_p stored in the capacitor

we have : $E_p = \frac{1}{2} Q\, U$

$$\Rightarrow \boldsymbol{E_p = \frac{Q^2}{4\pi\, \varepsilon_0\, \varepsilon_r}\left(\frac{R_2 - R_1}{R_1 R_2}\right)}$$

9.3. Cylindrical capacitor مكثفة اسطوانية

The cylindrical capacitor is made up of two coaxial and conductive cylinders, separated by an insulator of permittivity ε_0.

1) The electrostatic field $\vec{E}$ between the cylinders

T of Gauss : $\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}$

$R_1 < r < R_2 : Q_{int} = +Q \Rightarrow E.2\,\pi\, r\, h = \frac{+Q}{\varepsilon_0}$

$$\Rightarrow \boldsymbol{E = \frac{Q}{2\,\pi\varepsilon_0\;H\;r}}$$

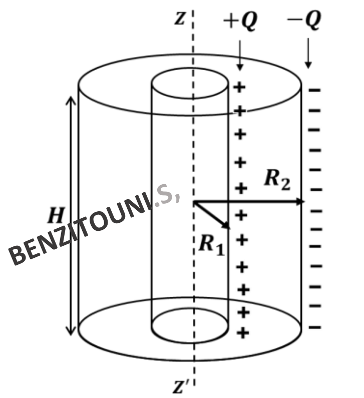


Figure.12. Cylindrical capacitor

2) The potential difference U between the cylinders.

On a : $U = V_1 - V_2$ et $\vec{E} = -\overrightarrow{grad}\, V \Rightarrow dV = -E\, dr$

$$\Rightarrow \boldsymbol{U = \frac{Q}{2\pi\, \varepsilon_0 H}\, ln\, \frac{R_2}{R_1}}$$

3) The capacitance C

we have : $C = \frac{Q}{U}$

$$\Rightarrow \boldsymbol{C = \frac{2\pi\, \varepsilon_0 H}{ln\, \frac{R_2}{R_1}}}$$

4) Energy E_p stored in the capacitor

we have : $E_p = \frac{1}{2} Q\, U$

$$\Rightarrow \boldsymbol{E_p = \frac{Q^2}{2\pi\, \varepsilon_0\, H}\, ln\, \frac{R_2}{R_1}}$$

10. Association of capacitors جمع المكثفات

10.1. In series الربط على التسلسل

$$\begin{cases} U = U_1 + U_2 + U_3 \dots\dots\dots U_n \\ U = \dfrac{Q}{C} \end{cases}$$

$$\Rightarrow \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \dots\dots\dots \frac{Q}{C_n}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots\dots\dots \frac{1}{C_n}$$

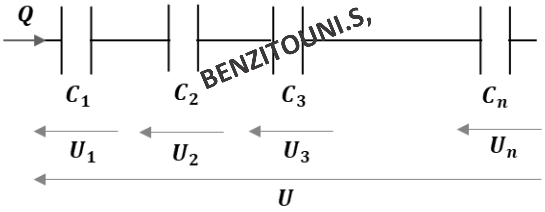


Figure.13. Association of capacitors in series

$$\boldsymbol{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

10.2. In parallel الربط على التفرع

$$\begin{cases} U = U_1 = U_2 = U_n \\ Q = Q_1 + Q_2 + Q_3 \dots\dots\dots Q_n \\ \quad Q = U.C \end{cases}$$

$$\Rightarrow U.C_{eq} = U.C_1 + U.C_2 + U.C_3 \dots\dots\dots U.C_n$$

$$\Rightarrow C_{eq} = C_1 + C_2 + C_3 \dots\dots\dots C_n$$

$$\boldsymbol{C_{eq} = \sum_{i=1}^{n} C_i}$$

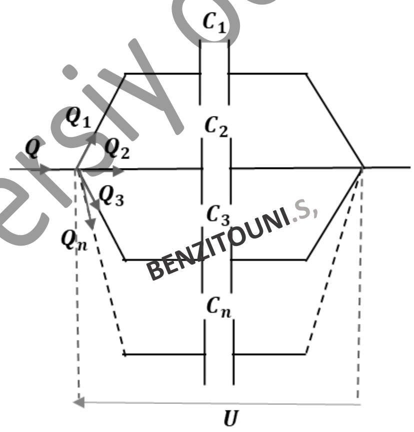
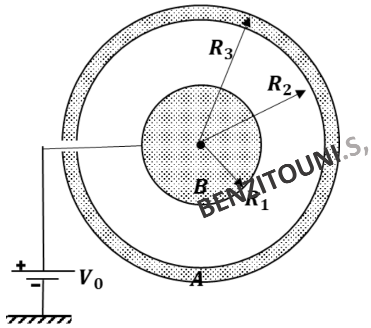


Figure.14. Association of capacitors in parallel

11. Exercises

Exercise-1 (The solution will be given in the course)

A hollow spherical conductor A, initially neutral, with interior radius $R_2 = 2R$ and exterior radius $R_3 = 4R$ surrounds a second spherical conductor B, with radius $R_1 = R$, brought to a potential V_0 via a generator. (See **figure** below). Conductor B carries a charge Q_0.



1) What are the charges carried by the interior and exterior surfaces of the conductor A. justify.
2) By applying Gauss's theorem, determine the expression of the electric field E in the following four regions:

$$r < R, \qquad R < r < 2R, \qquad 2R < r < 4R, \qquad r > 4R.$$

3) Considering that V_A is the potential of conductor A and knowing that the electric potential is zero at infinity, determine the expression of the electric potential in the four regions.
4) Deduce the charge Q_0 as a function of R, V_0 and ε_0 .