



Problem 1 Maximize the following

$$\begin{cases} \max xy \\ \text{subject to} \\ 4x^2 + y^2 = 36 \end{cases}$$

Problem 2 Minimize the following

$$\begin{cases} \min x^2 + y^2 \\ \text{subject to} \\ x^2y - 25 = 0 \end{cases}$$

Problem 3 Minimize the following

$$\begin{cases} \min f(x, y, z) = z \\ \text{subject to} \\ g_1(x, y, z) = x + y + z - 12 = 0 \\ g_2(x, y, z) = x^2 + y^2 - z = 0 \end{cases}$$

Problem 4 Let us show by an optimization calculation that the geometric mean of a set of positive values is always less than or equal to the arithmetic mean,

$$\sqrt[n]{x_1x_2\dots x_n} \leq \frac{x_1 + x_2 + \dots + x_n}{n}$$

Problem 5 Solve the problem

$$\begin{cases} \min \langle Ax, x \rangle \\ \text{st} \\ \|x\| = 1 \end{cases}$$

A is a symmetric matrix and $x = (x_1, x_2, \dots, x_n)^T$

Problem 6 We consider the function

$$f(x, y) = x^2 + y^2 - 4xy$$

subject to the constraint

$$x^2 + y^2 = 16.$$

What are the extrema of this function?

Problem 7 We consider the function

$$f(x, y) = x^3 + y^3$$

subject to the constraint

$$x^2 + y^2 = 4.$$

What are the extrema of this function?

Problem 8 We consider the function

$$f(x, y, z) = (x - 2)^2 + y^2 + z^2$$

subject to the constraint

$$x^2 + 2y^2 + 3z^2 = 4$$

What are the extrema of this function?

Problem 9 We consider the function

$$f(x, y, z) = x^3 + y + \frac{1}{2}z^2$$

subject to the constraints

$$x + y + z = 0$$

$$x + y - z = 0$$

What are the extrema of this function?

Problem 10 Consider the function

$$f(x, y, z) = \pi x^2 y$$

subject to the constraint

$$4\pi x^2 + 2\pi xy - \alpha = 0 = 0$$

Find the maximum of this function?

Problem 11 Consider the minimization problem with linear equality constraints (again $Q > 0$ and A has full row rank):

minimize $\frac{1}{2}x^T Q x$ subject to $Ax = b$

Deduce the solution of the following problem

minimize $\|x\|$ subject to $Ax = b$