Master 01

TD 01

Optimization

Problem 1 *Define the following*

$$\begin{cases} \max_{x \in R^2} u(x,y) \\ \textit{subject to} \\ x+y \leq 2 \end{cases}$$

$$\begin{cases} \min x+y+z^2 \\ \textit{subject to} \\ x^2+y^2+z^2=1 \\ \textit{and} \\ y=0 \end{cases}$$

$$\begin{cases} \textit{opt } xyz+z \\ \textit{subject to} \\ x^2+y^2+z \leq 6 \\ \textit{and} \\ y \geq 0, x \geq 0, z \geq 0 \end{cases}$$

Problem 2 *Write and convert the following problems from mixed to equality or inequality problems*

1- Find the maximum value of $f(x,y) = x^2y$*, if* x *and* y *are restricted to positive real numbers for which* $6x+5y=45$

2- Find the smallest value of $f(x,y) = 5x\cos(x+y)+16xy+21$*, over positive* x*.*

3- Find the smallest value of $f(x,y,z) = x^2+y^3+3\sin(z)$*, over positive* $x+y \geq z-1$ *and* $y+z=0$*.*

Problem 3 *Give the types of the following problems*

1-Objective function: Maximize $Z = 5X + 2Y$

Subject to:

$2X + Y \leq 100$
$X + 3Y \leq 90$
$X, Y \geq 0$
2-Objective function: Minimize $Z = 13X + 2Y$
Subject to:
$2X + 3Y \leq 10$
X, Y *are integers*
3-Objective function: Minimize $Z = X^2 + Y^2$
Subject to:
$X + Y \geq 9$
$X, Y \geq 0$
3-Objective function: Minimize $Z = X^2 + Y^2$
Subject to:
$X^2 + Y^2 \geq 1$
$X, Y \geq 0$

Problem 4 *Find the formulas of the following*

1-For a rectangle whose perimeter is 20 m, find the dimensions that will maximize the area.

2- Suppose a consumer consumes two goods, x *and* y *and has utility function* $U(x, y) = xy$.

He has a budget of $400Da$. *The price of* x *is* $P_x = 10$ *and the price of* y *is* $P_y = 20$

3- Find the point on the parabola that is closest to the point (3,2). (Estimate the solution to the cubic equation which results)

4- You are tasked with building a can that holds 1 liter of liquid. To maximize profit, you must build the can such that the material used to build it is minimized.

What is the minimum surface area of the can required?