

## 2<sup>nd</sup> Written Examination

**Exercise A:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is even.

**Exercise B:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is odd.

**Exercise C:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is greater than 10.

**Exercise D:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is multiple of 3.

**Exercise E:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is multiple of 2 or multiple of 3.

**Solutions of Exercise A:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is even.

1° Truth table:

a	b	c	d	F	Type of term	Term
0	0	0	0	1	MinTerm	$\bar{a}\bar{b}\bar{c}\bar{d}$
0	0	0	1	0	MaxTerm	$a+b+c+d$
0	0	1	0	1	MinTerm	$\bar{a}\bar{b}cd$
0	0	1	1	0	MaxTerm	$a+b+\bar{c}+\bar{d}$
0	1	0	0	1	MinTerm	$\bar{a}bc\bar{d}$
0	1	0	1	0	MaxTerm	$a+b\bar{c}+d$
0	1	1	0	1	MinTerm	$\bar{a}bcd$
0	1	1	1	0	MaxTerm	$a+b\bar{c}+\bar{d}$

1	0	0	0	1	MinTerm	$ab\bar{c}\bar{d}$
1	0	0	1	0	MaxTerm	$\bar{a}+b+c+d$
1	0	1	0	1	MinTerm	$ab\bar{c}d$
1	0	1	1	0	MaxTerm	$\bar{a}+b+\bar{c}+d$
1	1	0	0	1	MinTerm	$abc\bar{d}$
1	1	0	1	0	MaxTerm	$\bar{a}+\bar{b}+c+d$
1	1	1	0	1	MinTerm	$abcd$
1	1	1	1	0	MaxTerm	$\bar{a}+\bar{b}+\bar{c}+d$

**2° Canonical forms:**

1<sup>st</sup> canonical form: Sum of Minterms:

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}\bar{d} + \bar{a}bc\bar{d} + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + abcd$$

2<sup>nd</sup> canonical form: Product of Maxterms:

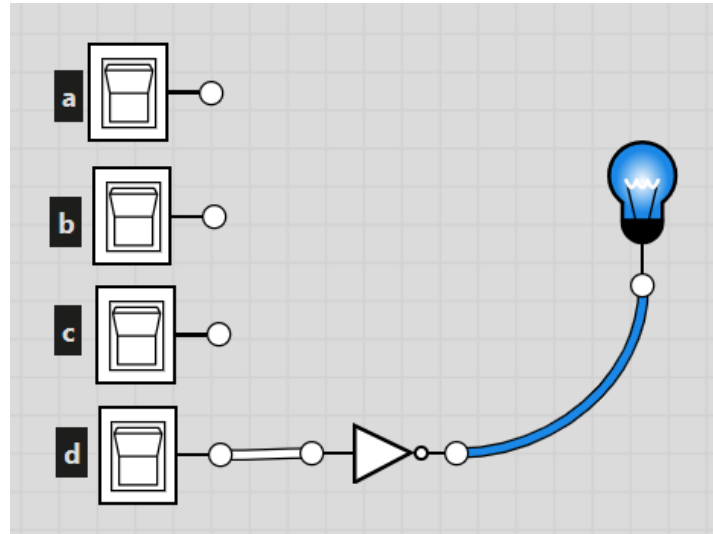
$$F(a,b,c,d) = (a+b+c+d)(a+b+\bar{c}+d)(a+\bar{b}+c+d)(a+\bar{b}+\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+d)$$

**3° Simplification:**

		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	0	0	1

$$F(a,b,c,d) = \bar{d}$$

**4° Logigram:**



**Solutions of Exercice B:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is odd.

**1° Truth table:**

a	b	c	d	F	Type of term	Term
0	0	0	0	0	MaxTerm	$a+b+c+d$
0	0	0	1	1	MinTerm	$\bar{a}\bar{b}\bar{c}d$
0	0	1	0	0	MaxTerm	$a+b+\bar{c}+d$
0	0	1	1	1	MinTerm	$\bar{a}\bar{b}cd$
0	1	0	0	0	MaxTerm	$a+\bar{b}+c+d$
0	1	0	1	1	MinTerm	$\bar{a}b\bar{c}d$
0	1	1	0	0	MaxTerm	$a+\bar{b}+\bar{c}+d$
0	1	1	1	1	MinTerm	$\bar{a}bcd$
1	0	0	0	0	MaxTerm	$\bar{a}+b+c+d$
1	0	0	1	1	MinTerm	$a\bar{b}\bar{c}d$

1	0	1	0	<b>0</b>	<b>MaxTerm</b>	$\bar{a}+b+\bar{c}+d$
1	0	1	1	<b>1</b>	<b>MinTerm</b>	$ab\bar{c}d$
1	1	0	0	<b>0</b>	<b>MaxTerm</b>	$\bar{a}+\bar{b}+c+d$
1	1	0	1	<b>1</b>	<b>MinTerm</b>	$ab\bar{c}d$
1	1	1	0	<b>0</b>	<b>MaxTerm</b>	$\bar{a}+\bar{b}+\bar{c}+d$
1	1	1	1	<b>1</b>	<b>MinTerm</b>	$abcd$

**2° Canonical forms:**

1<sup>st</sup> canonical form: Sum of Minterms:

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abcd + ab\bar{c}\bar{d} + abcd$$

2<sup>nd</sup> canonical form: Product of Maxterms:

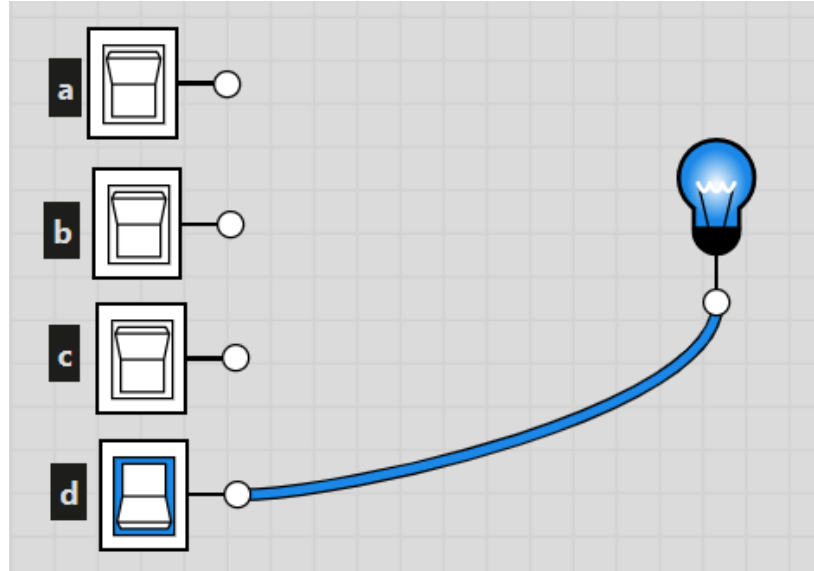
$$F(a,b,c,d) = (a+b+c+d)(a+b+\bar{c}+d)(a+\bar{b}+c+d)(a+\bar{b}+\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+d)$$

**3° Simplification:**

		cd			
		00	01	11	10
ab	00	0	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	1	1	0

$$F(a,b,c,d) = d$$

**4° Logigram:**



**Solution of Exercice C:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is is greater than 10.

**1° Truth table**  $(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)$

a	b	c	d	F	Type of term	Term
0	0	0	0	0	MaxTerm	$a+b+c+d$
0	0	0	1	0	MaxTerm	$a+b+c+d'$
0	0	1	0	0	MaxTerm	$a+b+c'+d$
0	0	1	1	0	MaxTerm	$a+b+c'+d'$
0	1	0	0	0	MaxTerm	$a+b'+c+d$
0	1	0	1	0	MaxTerm	$a+b'+c+d'$
0	1	1	0	0	MaxTerm	$a+b'+c'+d$

0	1	1	1	0	MaxTerm	$a+b\bar{c}+\bar{d}$
1	0	0	0	0	MaxTerm	$\bar{a}+b+c+d$
1	0	0	1	0	MaxTerm	$\bar{a}+b+c+\bar{d}$
1	0	1	0	0	MaxTerm	$\bar{a}+b+\bar{c}+d$
1	0	1	1	1	MinTerm	$a\bar{b}cd$
1	1	0	0	1	MinTerm	$abc\bar{d}$
1	1	0	1	1	MinTerm	$abc\bar{d}$
1	1	1	0	1	MinTerm	$abcd\bar{d}$
1	1	1	1	1	MinTerm	$abcd$

**2° Canonical forms:**

1<sup>st</sup> canonical form: Sum of Minterms:

$$F(a,b,c,d) = a\bar{b}cd + abc\bar{d} + abc\bar{d} + abc\bar{d} + abcd$$

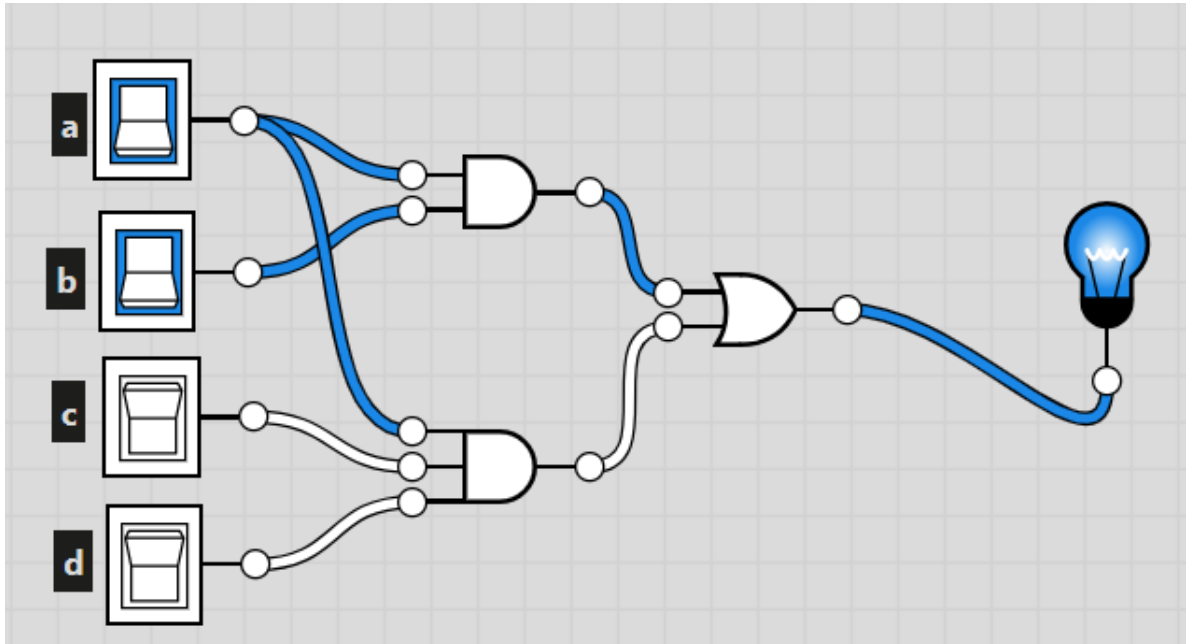
2<sup>nd</sup> canonical form: Product of Maxterms:

$$F(a,b,c,d) = (a+b+c+d)(a+b+c+d)(a+b+\bar{c}+d)(a+b+\bar{c}+d)(a+b\bar{c}+d)(a+b\bar{c}+d)(a+b\bar{c}+d)(a+b\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)$$

**3° Simplification:**

		cd			
		00	01	11	10
ab	00	0	0	0	0
	01	0	0	0	0
	11	1	1	1	1
	10	0	0	1	0

$$F(a,b,c,d) = ab + acd$$



**Solution of Exercice D:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is multiple of 3.

1<sup>o</sup> Truth table

a	b	c	d	F	Type of term	Term
0	0	0	0	1	MinTerm	$\bar{a}\bar{b}\bar{c}\bar{d}$
0	0	0	1	0	MaxTerm	$a+b+c+d$
0	0	1	0	0	MaxTerm	$a+b+\bar{c}+d$
0	0	1	1	1	MinTerm	$\bar{a}\bar{b}cd$
0	1	0	0	0	MaxTerm	$a+b\bar{c}+d$
0	1	0	1	0	MaxTerm	$a+b\bar{c}+d$
0	1	1	0	1	MinTerm	$\bar{a}bcd$
0	1	1	1	0	MaxTerm	$a+b\bar{c}+d$
1	0	0	0	0	MaxTerm	$\bar{a}+b+c+d$

1	0	0	1	1	MinTerm	$ab\bar{c}d$
1	0	1	0	0	MaxTerm	$\bar{a}+b+\bar{c}+d$
1	0	1	1	0	MaxTerm	$\bar{a}+b+\bar{c}+\bar{d}$
1	1	0	0	1	MinTerm	$abc\bar{d}$
1	1	0	1	0	MaxTerm	$\bar{a}+b^{-}+c+d$
1	1	1	0	0	MaxTerm	$\bar{a}+b^{-}+\bar{c}+d$
1	1	1	1	1	MinTerm	$abcd$

**2° Canonical forms:**

1<sup>st</sup> canonical form: Sum of Minterms:

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abc\bar{d} + abcd$$

2<sup>nd</sup> canonical form: Product of Maxterms:

$$F(a,b,c,d) = (a+b+c+d)(a+b^{-}+c+d)(a+b^{-}+c+\bar{d})(a+b^{-}+\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+b+c+d)$$

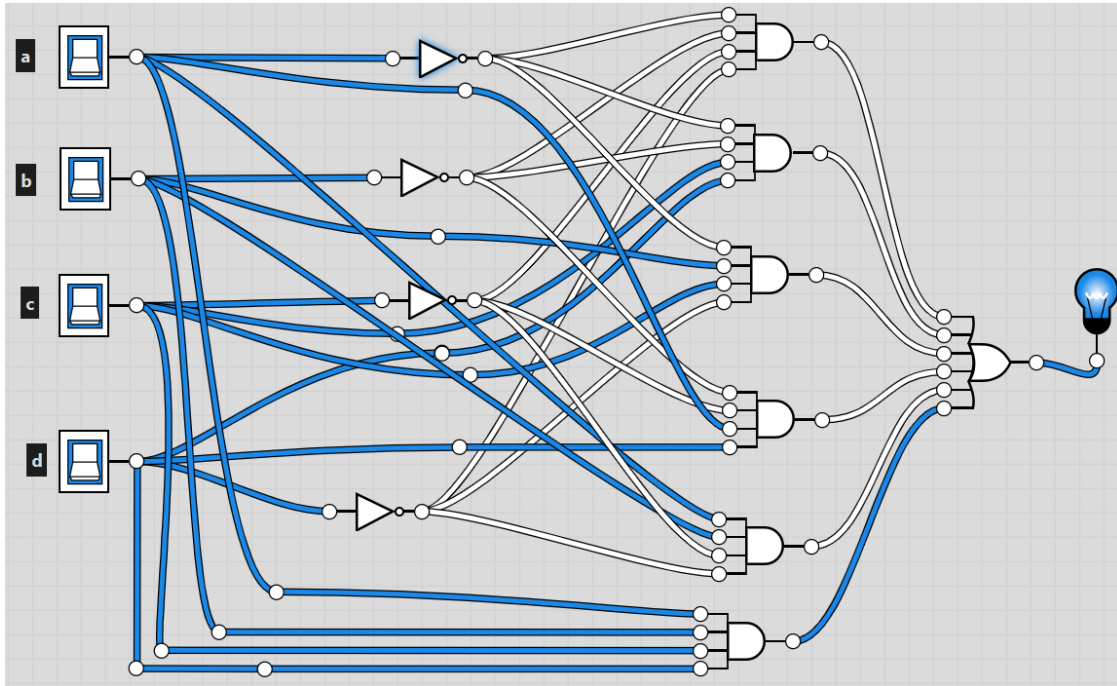
**3° Simplification:**

		cd			
		00	01	11	10
ab	00	1	0	1	0
	01	0	0	0	1
	11	1	0	1	0
	10	0	1	0	0

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abc\bar{d} + abcd$$



**4° Logigram:**



**Solution of Exercice E:** Create a logic circuit to check whether a four-digit (a,b,c and d) binary number is multiple of 2 or multiple of 3.

**1° Truth table**

a	b	c	d	F	Type of term	Term
0	0	0	0	1	MinTerm	$a\bar{b}\bar{c}\bar{d}$
0	0	0	1	0	MaxTerm	$a+b+c+d$
0	0	1	0	1	MinTerm	$a\bar{b}c\bar{d}$
0	0	1	1	1	MinTerm	$a\bar{b}cd$
0	1	0	0	1	MinTerm	$a\bar{b}c\bar{d}$
0	1	0	1	0	MaxTerm	$a+b\bar{c}+d$
0	1	1	0	1	MinTerm	$a\bar{b}cd$

0	1	1	1	0	MaxTerm	$a+b\bar{c}+d$
1	0	0	0	1	MinTerm	$ab\bar{c}\bar{d}$
1	0	0	1	1	MinTerm	$ab\bar{c}d$
1	0	1	0	1	MinTerm	$ab\bar{c}\bar{d}$
1	0	1	1	0	MaxTerm	$\bar{a}+b+c+d$
1	1	0	0	1	MinTerm	$abc\bar{d}$
1	1	0	1	0	MaxTerm	$\bar{a}+b\bar{c}+d$
1	1	1	0	1	MinTerm	$abcd$
1	1	1	1	1	MinTerm	$abcd$

**2° Canonical forms:**

1<sup>st</sup> canonical form: Sum of Minterms:

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}\bar{d} + \bar{a}bc\bar{d} + \bar{a}bcd + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + abcd + abcd$$

2<sup>nd</sup> canonical form: Product of Maxterms:

$$F(a,b,c,d) = (a+b+c+d)(a+b\bar{c}+d)(a+b\bar{c}+d)(\bar{a}+b+c+d)(\bar{a}+b\bar{c}+d)$$

**3° Simplification:**

		cd			
		00	01	11	10
ab	00	1	0	1	1
	01	1	0	0	1
	11	1	0	1	1
	10	1	1	0	1

$$F(a,b,c,d) = \bar{d} + ab\bar{c} + abc + \bar{a}b\bar{c}$$

**4° Logigram:**

