Boolean Algebra

Exercise 1: Draw the truth table of the following expressions:

A	B	AB	$\overline{AB}$	$F(A,B)=A+\overline{AB}$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

A	B	AB	$\overline{AB}$	A+B	$F(A,B)=\overline{AB}(A+B)$
0	0	0	1	0	0
0	1	0	1	1	1
1	0	0	1	1	1
1	1	1	0	1	0

A	B	C	$\overline{B}$	$\overline{C}$	ABC	$AB\overline{C}$	$A\overline{B}C$	$A\overline{BC}$	$F(A,B,C)=ABC+AB\overline{C}+A\overline{B}C+A\overline{BC}$
0	0	0	1	1	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0
1	0	0	1	1	0	0	0	1	1
1	0	1	1	0	0	0	1	0	1
1	1	0	0	1	0	1	0	0	1
1	1	1	0	0	1	0	0	0	1

Exercise 2: Prove the following theorems by the truth table

1 Idempotence : $a+a+a+....=a$

a	a	a		$a+a+a+a+a+a+a$	$a.a.a.a.a$
0	0	0		0	0
1	1	1		1	1

2 Identity $a+0=a$
$a.1=a$

a	0	1		a+0	a.1
0	0	1		0	0
1	0	1		1	1

3 Absorption $a.0=0$
$a+1=1$

a	0	1		$a.0$	$a+1$
0	0	1		0	1
1	0	1		0	1

4 Complementary $a + \overline{a} = 1$
$a.\overline{a} = 0$

a		$a+\overline{a}$	$a.\overline{a}$
0		1	0
1		1	0

$\overline{a.b} = \overline{a} + \overline{b}$

a	b	'a	b'		a.b	$\overline{a.b}$	$\overline{a}+\overline{b}$
0	0	1	1		0	1	1
0	1	1	0		0	1	1
1	0	0	1		0	1	1
1	1	0	0		1	0	0

$\overline{a+b} = \overline{a}.\overline{b}$

a	b	'a	b'	a+b	$\overline{a+b}$	$\overline{a}.\overline{b}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

Exercise 3: Prove the following equations using the properties of Boolean algebra:

1° a+a.b= a. (1+b) (common factors)
= a.1 (absorption)
= a (identity)

2° a.(a+b)= a.a + a.b (distribution of . over +)
= a+ a.b (idempotence)
= a.(1+b) (common factors)
= a.1 (absorption)
= a (identity)

3°

$a+\overline{a}.b = (a+\overline{a}) . (a+b)$ (distribution of + over .)
= 1. (a+b)
= (a+b)

4°

$$(a+b).(a+\bar{b}) = a+b\bar{b}$$
$$= a$$

Exercise 4:

1° $(a+b)(a+c) = a+(b.c)$ (distribution of + over .)

2°

$$(a+b).(\bar{a}+c) = a.\bar{a}+a.c+\bar{a}b+b.c$$
$$=0+ac+\bar{a}b+b.c\,(a+\bar{a})$$
$$=ac+\bar{a}b+abc+\bar{a}bc$$
$$=\bar{a}b(1+c)+ac(1+b)$$
$$=\bar{a}b+ac$$

3°

$$\overline{\bar{a}.b+\overline{\bar{a}+b}}$$

$$\begin{aligned}
&\overline{\bar{a}.b+\overline{\bar{a}+b}}\\
&=(\overline{\bar{a}.b}).(\overline{\overline{\bar{a}+b}})\\
&=(\bar{\bar{a}}+\bar{b}).(\bar{a}+b)\\
&=(a+\bar{b})(\bar{a}+b)\\
&=a.\bar{a}+a.b+\bar{a}.\bar{b}+b.\bar{b}\\
&=a.b+\bar{a}.\bar{b}
\end{aligned}$$

Exercise 5:

$\boxed{1}$ $f1(x,y,z) = xy + x\overline{z} + \overline{y}z$

x	y	z		f1	Minterm	Maxterm
0	0	0		0		$(x+y+z)$
0	0	1		1	$\overline{x}.\overline{y}z$	
0	1	0		0		$(x+.\overline{y}+z)$
0	1	1		0		$(x+.\overline{y}+.\overline{z})$
1	0	0		1	$x.\overline{y}.\overline{z}$	
1	0	1		1	$x.\overline{y}z$	
1	1	0		1	$xy.\overline{z}$	
1	1	1		1	xyz	

1st canonical form:
$F1 = \overline{x}.\overline{y}z + x.\overline{y}.\overline{z} + x.\overline{y}z + xy.\overline{z} + xyz$
2nd canonical form
$F1 = (x+y+z)\ \ (x+\overline{y}+z)(x+\overline{y}+\overline{z})$

$\boxed{2}$ F2(a, b, c) = 1 if the number of variables at 1 is even

a	b	c		f2	Minterm	Maxterm
0	0	0		1	$\overline{a}\overline{b}\overline{c}$	
0	0	1		0		$(a+b+\overline{c})$
0	1	0		0		$(a+\overline{b}+c)$
0	1	1		1	$\overline{a}bc$	
1	0	0		0		$(\overline{a}+b+c)$
1	0	1		1	$a\overline{b}c$	
1	1	0		1	$ab\overline{c}$	
1	1	1		0		$(\overline{a}+\overline{b}+\overline{c})$

1st canonical form
F2= $\overline{a}.\overline{b}.\overline{c} + \overline{a}bc + a.\overline{b}c + ab.\overline{c}$
2nd canonical form
F2= $(a+b+\overline{c})(a+\overline{b}+c)(\overline{a}+b+c)(\overline{a}+\overline{b}+\overline{c})$

a	b	c	d		f3	Minterm	Maxterm
0	0	0	0		0		$(a+b+c+d)$
0	0	0	1		0		$(a+b+c+\overline{d})$
0	0	1	0		0		$(a+b+\overline{c}+d)$
0	0	1	1		1	$\overline{a}\overline{b}cd$	
0	1	0	0		0		$(a+\overline{b}+c+d)$
0	1	0	1		1	$\overline{a}b\overline{c}d$	
0	1	1	0		1	$\overline{a}bc\overline{d}$	
0	1	1	1		1	$\overline{a}bcd$	
1	0	0	0		0		$(\overline{a}+b+c+d)$
1	0	0	1		1	$a\overline{b}\overline{c}d$	
1	0	1	0		1	$a\overline{b}c\overline{d}$	
1	0	1	1		1	$a\overline{b}cd$	
1	1	0	0		1	$ab\overline{c}\overline{d}$	
1	1	0	1		1	$ab\overline{c}d$	
1	1	1	0		1	$abc\overline{d}$	
1	1	1	1		1	$abcd$	

1st canonical form
$F3 = \overline{a}\overline{b}cd + \overline{a}b\overline{c}d + \overline{a}bc\overline{d} + \overline{a}bcd + a\overline{b}\overline{c}d + a\overline{b}c\overline{d} + a\overline{b}cd + ab\overline{c}\overline{d} + ab\overline{c}d + abc\overline{d} + abcd$

2nd Canonical form

$F3 = (a+b+c+d)(a+b+c+\overline{d})(a+b+\overline{c}+d)(a+\overline{b}+c+d)(\overline{a}+b+c+d)$

Exercice 6:

$\boxed{1}$ $f1(x,y,z) = xy + x\overline{z} + \overline{y}z$

x \ yz	00	01	11	10
0	0	1	0	0
1	1	1	1	1

$f_1 = x + \overline{y}z$

$\boxed{2}$ f2(a, b, c) = 1 if the count of variables at 1 is even

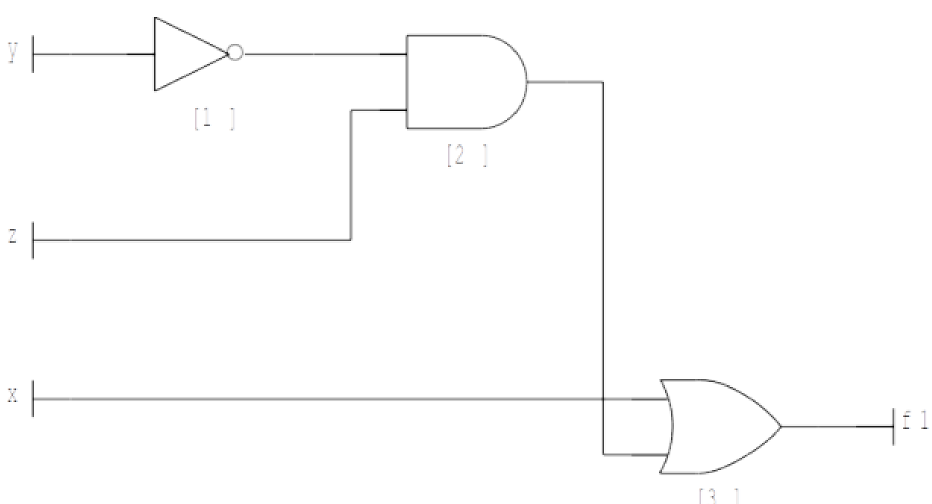
a \ bc	00	01	11	10
0	1	0	1	0
1	0	1	0	1

$$f_2 = \overline{abc} + a\overline{b}c + \overline{a}bc + ab\overline{c}$$

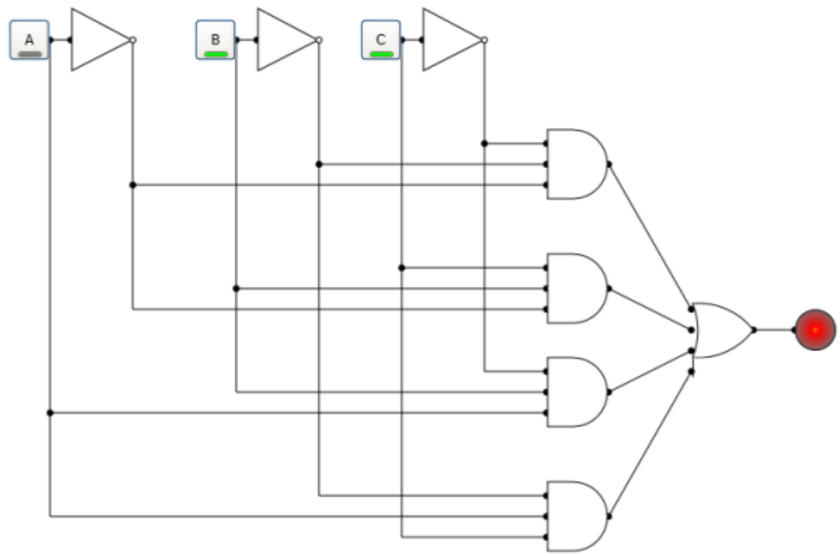
$\boxed{3}$ f3(a, b, c , d) = 1 if at least two variables are equal to 1

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

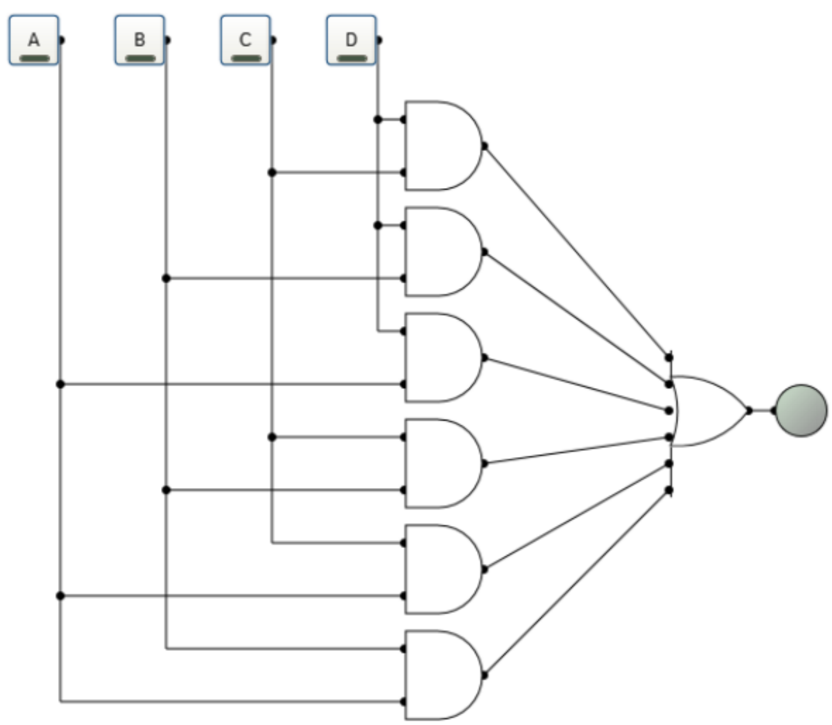
Exercice 7:



Logigramme de la fonction $f1(x, y, z) = xy + x\overline{z} + \overline{y}z$.



Logigram of function f2(a, b, c) = 1 if the count of variables at 1 is even.



Logigram of function f3(a, b, c , d) = 1 if at least two variables are equal to 1.

Exercice 8:

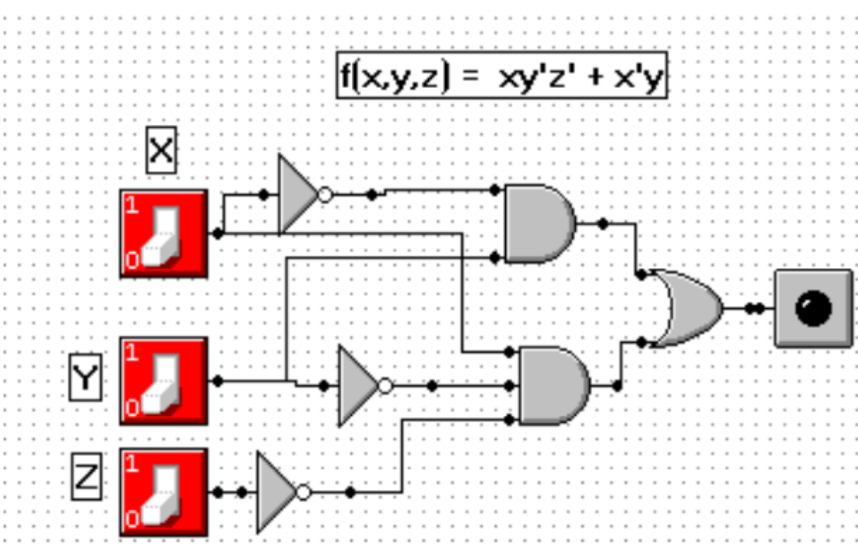
Truth table:

x	y	z		f4
0	0	0		0
0	0	1		0
0	1	0		1
0	1	1		1
1	0	0		1
1	0	1		0
1	1	0		0
1	1	1		0

Canonical forms:
1st canonical form:
$F4(x,y,z) = \overline{x}.y.\overline{z} + \overline{x}.y.z + x.\overline{y}.\overline{z}$
2nd canonical form:
$F4(x,y,z) = (x+y+z)(x+y+\overline{z})(x+\overline{y}+z)(\overline{x}+\overline{y}+z)(\overline{x}+\overline{y}+\overline{z})$

x \ yz	00	01	11	10
0	0	0	1	1
1	1	0	0	0

Simplification:
$f(x,y,z) = x.\overline{y}.\overline{z} + \overline{x}.y$



Logigram of $F(x,y,z) = x \oplus (y+z)$.