Exercises section 1

Exercise 1 (ordinary differential equations (ODEs) using eigenvalues and eigenvectors)

Exercise 1.1: Solving a 2x2 System of Linear ODEs

$$\begin{cases} \dfrac{dx}{dt} = x(t) = 4x - y \\ \dfrac{dy}{dt} = y(t) = 2x + y \end{cases}$$

1. Find the eigenvalues of the matrix A.
2. Find the corresponding eigenvectors.
3. Write the general solution in terms of the eigenvalues and eigenvectors.

Exercise 2 (ordinary differential equations (ODEs) using eigenvalues and eigenvectors)

Exercise 1.1: Solving a 3x3 System of Linear ODEs

Consider the following systems:

$$S_1 = \begin{cases} x(t) = 2x - y + z \\ y(t) = 3x + y - z \\ z(t) = 4z \end{cases} ; S_2 = \begin{cases} x(t) = x - 4y + 2z \\ y(t) = 2x - 3y + z \\ z(t) = x - 2y \end{cases} ; S_3 = \begin{cases} x(t) = 4x - 2y - z \\ y(t) = x + 3y - z \\ z(t) = 2y + 5z \end{cases}$$

1. Write the General Solution of ODE system S_1.
2. Do the same and Write the General Solution S_2.
3. On System S_3 Assume we have the initial condition $X(t) = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ Determine Constants $C_1, C_2, and\ C_3$, then the final form of solution.

Exercise 3: Gaussian Elimination, Gauss-Jordan and Cholesky algorithm

Solve the following systems using Gaussian elimination, and determine if the system is consistent or inconsistent.

$$\begin{cases} 2x - 3y + z = 5 \\ 3x + 2y - 4z = 7 \\ x - y + 2z = 4 \end{cases} \qquad \begin{cases} 2x + 4y - 3z = 11 \\ 3x - 5y + 2z = 9 \\ 4x - y + 3z = 13 \end{cases} \qquad \begin{cases} 2x + 3y - z = 5 \\ 4x + 6y - 2z = 10 \\ 6x + 9y - 3z = 15 \end{cases}$$

If a system cannot be solved using **Gaussian elimination**, attempt to find the closest solution with a stopping error ε=0.01 using the **Jacobi method**. Assume the following initial guess: $X(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$..

Consider the following two systems of equations:

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$$S_1 \begin{cases} -9\,x - 6\,y + 15\,z = 93 \\ -6\quad x + 12\,y + 14\,z = -2 \\ 15\,x + 14\,y\ - 25\,z = -179 \end{cases} \qquad S_2 \begin{cases} 8x - 3\,y + 2\,z = 8 \\ 4\,x + 11\,y - z = 23 \\ 6\,x + 3\,y\ + 12\,z = 48 \end{cases}$$

1. Solve S_1 using the **Cholesky algorithm**.
2. Solve S_2 using the **Jacobi algorithm** with the initial guess: $X(0) = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \varepsilon = 10^{-1}$.