CHAPTER 2

Number Systems

2.1. Introduction

Different information such as numbers, text, images, sounds, video, programs, etc. are processed by computers. This information is always represented in binary form, a series of 0s and 1s.

2.2. Definition

A number system is the method of representing numbers using a set of symbols called digits, such that the number of symbols used determines the number base.

Exemple :

- ❖ In the decimal system, we use a set of 10 symbols which are: {0,1,2,...,9}.
- ❖ In the decimal system, the number $(1515.2)_{10}$ is written as: :

  $(1515,2)_{10}= 1\times10^3+5\times10^2+1\times10+5\times10^0+2\times10^{-1}$

2.3. Presentation of decimal, binary, octal and hexadecimal systems

General rule

Let the base **b** be made up of **b** symbols {S0, S1, ..., Sb-1}, we can write the number N in the base **b** as follows:

$N= (a_n\, a_{n-1} \ldots a_1 a_0,\, a_{-1} a_{-2} \ldots a_{-m})_b$

⇨ $\boxed{N=a_n\times b^n+a_{n-1}\times b^{n-1}+\ldots+ a_0\times b^0+a_{-1}\times b^{-1}+ \ldots+a_{-m}\times b^{-m} \ldots}$ ....**(1)**

⇨ $N = \sum_{i=-m}^{n} a_i \times b_i$ $\qquad 0\leq a_i \leq b-1$

- ➢ The element $\mathbf{a_i}$ is the number whose rank is **i** and its weight is $\mathbf{b^i}$.
- ➢ The entire part is: $a_n\, a_{n-1} \ldots a_1 a_0$
- ➢ The fractional part is : $a_{-1} a_{-2} \ldots a_{-m}$

Example :

Let the number $(123,5)_{10}$

- ✓ The base is : 10 ;
- ✓ The entire part is : 123 ;
- ✓ The fractional part is : 5 ;
- ✓ The number 2: its rank is 1 and its weight 10.

The different number systems

a) The decimal system

It is a number system of including :

- ❖ The base is : **b=10** ;

- The set of symbols consists of 10 symbols : $a_i \in \{0, 1, 2, \dots, 9\}$

A number **N** is represented in this system (decimal) as follows:

$$(N)_8 = a_n\times10^n+a_{n-1}\times10^{n-1}+\dots+a_1\times10^1+a_0\times10^0+a_{-1}\times10^{-1}+\dots+a_{-m}\times10^{-m}$$

b) The binary system

It is a number system including:

- The base is : **b = 2** ;
- The symbol set consists of two symbols: $a_i \in \{0, 1\}$.

A number **N** is represented in this system (**binary**) as follows:

$$(N)_2 = a_n\times2^n+a_{n-1}\times2^{n-1}+\dots+a_1\times2^1+a_0\times2^0+a_{-1}\times2^{-1}+\dots+a_{-m}\times2^{-m}$$

c) The octal system

It is a number system including:

- The base is : **b=8** ;
- The symbol set consists of 8 symbols: $a_i \in \{0, 1, 2, \dots, 7\}$.

A number **N** is represented in this system (**octal**)) as follows:

$$(N)_8 = a_n\times8^n+a_{n-1}\times8^{n-1}+\dots+a_1\times8^1+a_0\times8^0+a_{-1}\times8^{-1}+\dots+a_{-m}\times8^{-m}$$

d) The hexadecimal system

It is a number system including:

- The base is : **b=16** ;
- The symbol set consists of 16 symbols: $a_i \in \{0, 1, \dots, 9, A, B, C, D, E, F\}$.

A number **N** is represented in this system (**hexadecimal**) as follows:

$$(N)_{16} = a_n\times16^n+a_{n-1}\times16^{n-1}+\dots+a_1\times16^1+a_0\times16^0+a_{-1}\times16^{-1}+\dots+a_{-m}\times16^{-m}$$

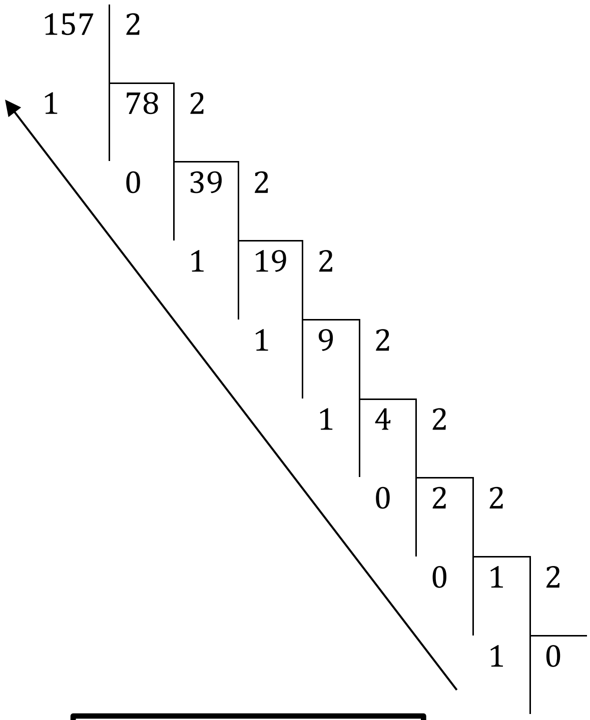
2.4. Conversion between these different systems

2.4.1. Transformation of a number from the decimal system to the binary system $(N)_{10} \rightarrow (N)_2$

The transformation of a decimal number into a number in the binary system is the operation of successive Euclidean (integer) division of the number **N** by **2** until the division result is equal to 0 and we keep the remainders of the operations of division and order them from the last remainder to the first remainder, that is to say the last remainder becomes the first and the first remainder becomes the last.

Example :

(157) $_{10}$ → (?)$_2$



⇨ $\boxed{(157)_{10} = (10011101)_2}$

❖ **<u>Transformation of the fractional part to the binary system</u>**

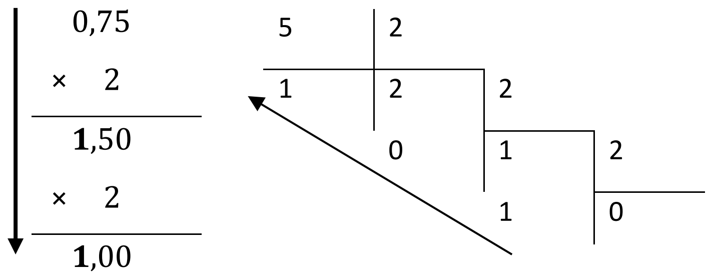
Let the number $(5.75)_{10}$,

To transform it into the binary system, we follow the following method:

⇨ $(5)_{10} = (101)_2$

⇨ $(0,75)_{10} = (0.11)_2$

⇨ $(5,75)_{10} = (101.11)_2$



⇨ $\boxed{(5,75)_{10} = (101.11)_2}$

<u>2.4.2. Transformation of a number from the binary system to the decimal system:</u> $(N)_2 \rightarrow (N)_{10}$

To transform a number written in the binary system to a number written in the decimal system we use the formula (rule) noted (1).

Example :

$(10011101)_2 = 1\times2^7+0\times2^6+0\times2^5+1\times2^4+1\times2^3+1\times2^2+0\times2^1+1\times2^0$

$\Longrightarrow (10011101)_2 = (157)_{10}$

$(11010011.1011) = 1\times2^7+1\times2^6+0\times2^5+1\times2^4+0\times2^3+0\times2^2+1\times2^1+1\times2^0+1\times2^{-1}+0\times2^{-2}+1\times2^{-3}+1\times2^{-4}$

$(11010011.1011) = (211{,}07421875)_{10}$

2.4.3. Transformation of a number from binary to octal system :

From binary representation, we can write the number in octal system as follows:

- For the entire part, the transformation is done by constituting groups of numbers, each group of which is made up of three numbers starting from the right.
- For the fractional part, the transformation is done by the constitution of groups of numbers, each group of which is composed of three numbers starting from the left.

Example : be the following numbers in the binary system:

$(10101.010)_2 = (\ ?)_8$

$(\underbrace{010}_{2}\underbrace{101}_{5}.\underbrace{010}_{2})_2 = (25.2)_8$

$(1100110111.0111101)_2 = (?)_8$

$(\underbrace{001}_{1}\underbrace{100}_{4}\underbrace{110}_{6}\underbrace{111}_{7}.\underbrace{011}_{3}\underbrace{110}_{6}\underbrace{100}_{4})_2 = (1467.364)_8$

2.4.4. Transformation of a number from binary to octal system:

The transition from the octal system to the binary system is done in the opposite way:

- Convert each digit of the octal number into its binary representation but on three binary digits.

Example :

Let the number $(17)_8 = (\ ?)_2$

$(1\ 7)_8 = (001111)_2$

001 111

$(3\quad 2\quad 1\quad 5)_8 = (011010001101)_2$

$(011\ \ 010\ \ 001\ \ 101)_2$

2.4.5. Transforming a number from decimal to octal system: $(N)_{10} \rightarrow (N)_8$

The transformation of the number directly from the decimal system to the octal system is done by the successive Euclidean division of the number by 8 until the result of the division is equal to 0 and we keep the remainders of the division operations and order them from the last remainder to the first remainder, i.e. the last remainder becomes the first and the first remainder becomes the last.

Example : $(225{,}732)_{10} = (\,?)_8$

225	8		
1	28	8	
	4	3	8
		3	0

$$
\begin{array}{r}
0{,}732 \\
\times \quad 8 \\
\hline
\mathbf{5}{,}856 \\
\times \quad 8 \\
\hline
\mathbf{6}{,}848 \\
\times \quad 8 \\
\hline
\mathbf{6}{,}784
\end{array}
$$

⇨ $(225{,}732)_{10}=(341.56662)_8$

2.4.6. Transformation from the octal to the decimal system: $(N)_8 \rightarrow (N)_{10}$

To transform a number written in the octal system to a number written in the decimal system, we use the formula (the rule) noted (1).

Example :

$(341)_8 = 3\times8^2 + 4\times8^1 + 1\times8^0 = (225)_{10}$

$(256.14)_8 = 2\times8^2 + 5\times8^1 + 6\times8^0 + 1\times8^{-1} + 4\times8^{-2} = (174{,}1875)_{10}$

2.4.7. Transformation from binary to hexadecimal system: $(N)_2 \rightarrow (N)_{16}$

From the binary representation, we can write the number in the hexadecimal system as:

- For the entire part, the transformation is done by the constitution of groups of numbers, each group of which is made up of four numbers, starting from the right.
- For the fractional part, the transformation is done by the constitution of groups of numbers, each group of which is composed of four numbers starting from the left.

Example :

$(\underbrace{0010}_{2}\underbrace{1110}_{E})_2 = (2E)_{16}$

$(\underbrace{0001}_{1}\underbrace{1010}_{A}.\underbrace{1010}_{A})_2=(1A.A)_{16}$

2.4.8. Transformation from hexadecimal to binary system: $(N)_{16} \rightarrow (N)_2$

The transition from the hexadecimal system to the binary system is done in the opposite way:

- Convert each digit of the hexadecimal number into its binary representation but over four binary digits.

Example :

$(1A.A)_{16} = (0001\ 1010.1010)_2$

0001 1010.1010

2.4.9. Transformation from decimal to hexadecimal system: $(N)_{10} \rightarrow (N)_{16}$

The transformation of the number directly from the decimal system to the hexadecimal system is done by the successive Euclidean division of the number by 16 until the result of the division is equal to 0 and we keep the remainders of the division operations and order them from the last remainder to the first remainder, i.e. the last remainder becomes the first and the first remainder becomes the last.

Example: Let the number $N=(189520{,}75)_{10}=(\ ?\ )_{16}$

Dividend	Divisor	Quotient	Remainder
189520	16	11845	**0**
11845	16	740	**5**
740	16	46	**4**
46	16	2	**14=E**
2	16	0	**2**

0,75

×16

12,00=**C**

$$\boxed{(189520{,}75)_{10} = (2E450.C)_{16}}$$

2.4.10. Transformation from hexadecimal to decimal system: $(N)_{16} \rightarrow (N)_{10}$

- The transformation of a number written in the hexadecimal system to a number written in the decimal system is done by applying the formula (rule) noted (1).

Example :

$(2E450.C)_{16} = 2\times16^4 + E\times16^3 + 4\times16^2 + 5\times16^1 + 0\times16^0 + C\times16^{-1}$

$= 2\times16^4 + 14\times16^3 + 4\times16^2 + 5\times16^1 + 0\times16^0 + 12\times16^{-1}$

$= (189520,75)_{10}$

Hexa (base 16)	Decimal (base 10)	Octal (base 8)	binary (base 2)
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111
10	16	20	10000

Table 2.1 : Correspondance décimal, binaire, octal, hexadécimal

2.5. Basic operations in the binary system:

a) Addition

To add two binary numbers, we proceed exactly as in decimal, but taking into account the following elementary addition table:

+	0	1
0	0	1
1	1	10

Examples :

```
  110101
+ 111100
--------
=1110001
```

```
 1011011
+0000111
--------
 1100010
+0000010
```

b) Subtraction:

In binary subtraction, we proceed as in decimal. When the quantity to be subtracted is greater than the quantity from which we are subtracting, we borrow 1 from the neighbor on the left. In binary, this 1 adds 2 to the quantity from which we subtract, while in decimal it adds 10.

Exemples :

```
 1101          1001011
-0111         -0101111
-----         --------
=0110         =0011100
```

c) Multiplication

Binary multiplication is performed like decimal multiplication. Here are the calculation rules to use:

×	0	1
0	0	0
1	0	1

Example:

$$\begin{array}{r} 1011 \\ \times\ 101 \\ \hline 1011 \\ 0000. \\ 1011.. \\ \hline 110111 \end{array}$$

d) Division :

Binary division is performed using subtractions and shifts, like decimal division, except that the digits of the quotient can only be 1 or 0. The quotient bit is 1 if the divisor can be subtracted, otherwise it is 0.

Example :

$$\begin{array}{r|l} 10101 & 11 \\ \cline{2-2} -11 & 111 \\ \cline{1-1} 100 & \\ -11 & \\ \cline{1-1} 0011 & \\ -\ \ 11 & \\ \cline{1-1} 00 & \end{array}$$