

Boolean Algebra

Exercise 1: Draw the truth table of the following expressions:

- $F_1(A, B) = A + \overline{AB}$
- $F_2(A, B) = \overline{AB} (A + B)$
- $F_3(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C}$

Exercise 2: Prove the following theorems by the truth table

Idempotence : $a + a + a + \dots = a$

Identity $a + 0 = a$ $a.1 = a$

Absorption $a.0 = 0$ $a + 1 = 1$

Complementary $a + \overline{a} = 1$ $a.\overline{a} = 0$

$\overline{a.b} = \overline{a} + \overline{b}$

$\overline{a + b} = \overline{a}.\overline{b}$

Exercise 3: Prove the following equations using the properties of Boolean algebra:

$$a + a.b = a$$

$$a.(a + b) = a$$

$$a + \overline{a}.b = a + b$$

$$(a + b)(a + \overline{b}) = a$$

Exercise 4: Simplify the following equations using the properties of Boolean algebra:

$$(a + b)(a + c)$$

$$(a + b)(\overline{a} + c)$$

$$\overline{\overline{a.b} + \overline{a + b}}$$

Exercise 5: Express the following functions in the first and second canonical form:

$$f1(x, y, z) = xy + x\overline{z} + \overline{y}z$$

$$f(a, b, c) = 1 \text{ if the count of variables at 1 is even}$$

$$f(a, b, c, d) = 1 \text{ if at least two variables are equal to 1}$$

Exercise 6: Simplify the functions of the previous exercise (exercice 5) using the Karnaugh map.

Exercise 7: Plot the logigrams of the functions of the previous exercise (exercice 5).

Exercise 8: Study the function: $F(x, y, z) = x \oplus (y + z)$