Continuous Random Variables and Probability distribution

* A discrete random variable (r.v) is one where whose possible values either constitute a finite set or else can be listed in an infinite sequence.
* A random variable whose set of possible values is an entire interval of numbers is not discrete.

Example: If a chemical compound is randomly selected and its PH X is determined, then X is a continuous rv because any PH value between 0 and 14 is possible.

Probability Distributions for Continuous variables (r.v)

Definition: Let X be a continuous r.v. Then a probability distribution or probability density function (pdf) of X is a function f(x) such that for any two numbers a and b with: $a \leq b$,

$$P(a \leq x \leq b) = \int_a^b f(x)dx$$

That is, the probability that X takes on a value in the ~~value~~ interval $[a,b]$ is the area above this interval and under the graph of the density function, as illustrated in Figure O-1

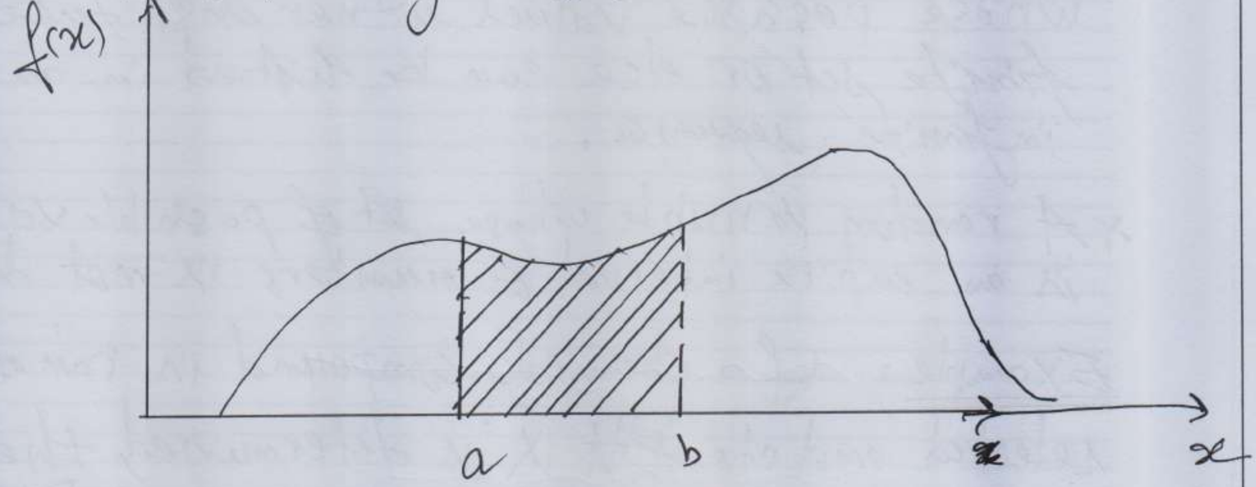


Figure O-1 $P(a \leq x \leq b)$.

For f to be pdf, it must satisfy the following two conditions:

① $f(x) \geq 0, \forall x$

② $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

<u>Example</u> Let $f(x)$ be defined as

$$f(x) = \begin{cases} 0{,}15\, e^{-0{,}15(x-0{,}5)} & , x \geq 0{,}5 \\ 0 & , x < 0{,}5 \end{cases}$$

We have: $f(x) \geq 0, \forall x \in \mathbb{R}$.

and $\int_{-\infty}^{+\infty} f(x)\,dx = \int_{0{,}5}^{+\infty} 0{,}5 \cdot e^{-0{,}15(x-0{,}5)}\,dx$ — (*)

$$(*) = \int_{0,5}^{+\infty} 0{,}15\, e^{+0{,}15\times 0{,}5} \cdot e^{-0{,}15x}\, dx$$

$$= 0{,}15\, e^{0{,}075} \int_{0,5}^{+\infty} e^{-0{,}15x}\, dx$$

$$= 0{,}15\, e^{0{,}075} \left(\frac{1}{-0{,}15}\, e^{-0{,}15x} \Big|_{0,5}^{+\infty} \right)$$

$$= 0{,}15\, e^{0{,}075} \left(0 - \frac{1}{-0{,}15}\, e^{-0{,}15\times 0{,}5} \right) = 1$$

Then $f(x)$ is a pdf

* We have: $p(X \leq 5) = \int_{-\infty}^{5} f(x)\,dx = \int 0{,}15\, e^{-0{,}15(x-0{,}5)}\, dx$

$$= 0{,}15\, e^{0{,}075} \int_{0,5}^{5} e^{-0{,}15x}\, dx = 0{,}15\, e^{0{,}075} \left(\frac{1}{-0{,}15}\, e^{-0{,}15x} \Big|_{0,5}^{5} \right)$$

$$= 0{,}15\, e^{0{,}075} \left(\frac{-1}{0{,}15}\, e^{-0{,}75} + \frac{1}{0{,}15}\, e^{-0{,}075} \right)$$

$$= 0{,}491$$

Cumulative Distribution Functions

Definition: The cumulative distribution function $F(x)$ for a continuous random variable is defined for every number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)\, dy$$

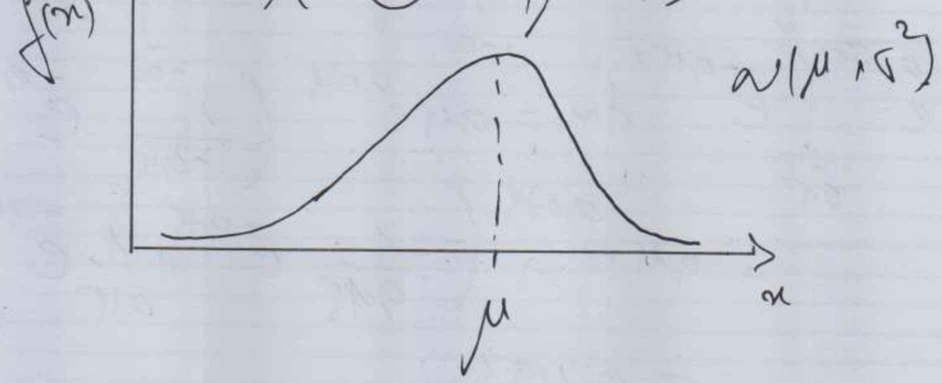
The normal Distribution : $\mathcal{N}(\mu, \sigma^2)$

A continuous random variable X is said to have a normal distribution with parameters μ and σ (or μ and σ^2), where $-\infty < \mu < +\infty$ and $0 < \sigma$ if the pdf of X is :

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

The statement that X is normally distributed with parameters μ and σ^2 is often abbreviated

$$X \sim \mathcal{N}(\mu, \sigma^2)$$



= (*)