University of Oum El Bouaghi

Faculty of Economic Sciences, Commercial Sciences and Management Sciences

First year, common trunk

Academic year : 2023-2024

Mathematics 1 Module
Solution of Series 05 (INTEGRALS).

Exercise 01: Find the antiderivatives of the following functions

3)

$$F(x)=-\frac{3}{4}x^4+\frac{5}{3}x^3-4x+k \quad ; \quad k\in\mathbb{R}$$

4)

$$F(x)=\frac{1}{5}x^5-\frac{1}{4}x^4+k \quad ; \quad k\in\mathbb{R}$$

5)

$$F(x)=\frac{-4}{x}+k; k\in\mathbb{R}$$

6)

$$F(x)=-x^{-1}+\frac{1}{2}x^{-2}+k.$$

7)

$$F(x)=\frac{1}{2}sin2x+\frac{1}{3}cos3x+k \quad ; \quad k\in\mathbb{R}$$

8)

$$F(x)=\frac{1}{4}cos^4x+k \quad ; \quad k\in\mathbb{R}$$

Exercise 02: 1)

$$f(x)=\frac{x+1}{(x^2+2x)^3}=\frac{1}{2}.\frac{2x+2}{(x^2+2x)^3}=\frac{1}{2}\frac{u'(x)}{u^3(x)}=\frac{1}{2}u'(x)u^{-3}(x)=\frac{1}{2}\times\frac{1}{-2}\times(-2)u'(x)u^{-3}(x),$$

$$u(x)=x^2+2x,\ n-1=-3,\ n=-2,\ F(x)=-\frac{1}{4}(x^2+2x)^{-2}=-\frac{1}{4(x^2+2x)^2}.$$

2)

$$f(x)=\frac{x}{x^2-1}=\frac{1}{2}\times\frac{2x}{x^2-1}=\frac{1}{2}\times\frac{u'(x)}{u(x)} \text{ avec } u(x)=x^2-1,\ F(x)=\frac{1}{2}\ln u(x)=\frac{1}{2}\ln(x^2-1)+k.$$

3)

$$f(x)=x-1+\frac{\ln x}{x}=x-1+\frac{1}{x}\times\ln x=x-1+\frac{1}{2}\times 2u'(x)\times u(x) \text{ avec } u(x)=\ln x,$$

$$F(x)=\frac{x^2}{2}-x+\frac{1}{2}u^2(x)=\frac{x^2}{2}-x+\frac{1}{2}(\ln x)^2+k.$$

Exercise 03: **1)**

$u'(t) = t \qquad u(t) = \frac{t^2}{2}$

$v(t) = \ln(t) \qquad v'(t) = \frac{1}{t}$

$I_1 = \left[\frac{t^2}{2}\ln(t)\right]_1^e - \int_1^e \frac{t}{2}dt$

$$I_1 = \frac{e^2}{2}\ln(e) - \frac{1^2}{2}\ln(1) - \left[\frac{t^2}{4}\right]_1^e = \frac{e^2}{2} - \frac{e^2-1}{4} = \frac{2e^2-e^2+1}{4} = \frac{e^2+1}{4}$$

2)

$I_2 = \int_0^{\frac{\pi}{2}} t\sin(t)\,dt$

$u'(t) = \sin(t) \qquad u(t) = -\cos(t)$

$v(t) = t \qquad v'(t) = 1$

$I_2 = [-t\cos(t)]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos(t)\,dt$

$$I_2 = -\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) + 0\times\cos(0) + [\sin(t)]_0^{\frac{\pi}{2}} = \sin\left(\frac{\pi}{2}\right) = 1$$

3)

$$\int xe^x\,dx$$

Solution: We will integrate this by parts, using the formula

$$\int f'g = fg - \int fg'$$

Let $g(x) = x$ and $f'(x) = e^x$ Then we obtain g' and f by differentiation and integration.

$f(x) = e^x$	$g(x) = x$
$f'(x) = e^x$	$g'(x) = 1$

$$\int f'g = fg - \int fg' \quad \text{becomes}$$

$$\int xe^x\,dx = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

Exercise 04: **1) Let** : $u = e^x$, **then**

$$\int_0^1 \frac{e^x dx}{\sqrt{e^x+1}} dx = \int_1^e \frac{du}{\sqrt{u+1}}$$
$$= \left[2\sqrt{u+1}\right]_1^e$$
$$= 2\sqrt{e+1} - 2\sqrt{2}$$

2) We have :

$$\int \frac{x}{\sqrt{9+4x^2}} dx = \int \frac{x}{\sqrt{9\left(1+\left(\frac{2x}{3}\right)^2\right)}} dx$$

Let :

$$t = \frac{2x}{3} \Leftrightarrow x = \frac{3}{2}t \Rightarrow dx = \frac{3}{2}dt$$

$$\int \frac{\frac{3}{2}t}{3\sqrt{1+t^2}} \times \frac{3}{2}dt = \frac{3}{4}\int \frac{t}{\sqrt{1+t^2}}dt = \frac{3}{4}\sqrt{1+t^2} + K = \frac{3}{4}\sqrt{1+\left(\frac{2x}{3}\right)^2} + K$$
$$= \frac{3}{4} \times \frac{1}{3}\sqrt{9+4x^2} + K = \frac{1}{8}\sqrt{9+4x^2} + K$$

3)

$$t = \sqrt{1-x} \Leftrightarrow t^2 = 1-x \Leftrightarrow x = 1-t^2 \Rightarrow dx = -2tdt$$
$$F(x) = \int -\frac{2t}{1+t}dt = -2\int \frac{t}{1+t}dt = -2\int \frac{t+1-1}{t+1}dt = -2\int \left(1-\frac{1}{t+1}\right)dt$$
$$= -2t + 2\ln(t+1) + K = -2\sqrt{1-x} + 2\ln\left(\sqrt{1-x}+1\right) + K$$