

Exercises (Series and partial derivatives)

I - Study the convergence of following series:

$$\textcircled{1} \sum_{k=1}^{+\infty} \frac{1}{2^k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

$$\textcircled{2} \sum_{n=2}^{+\infty} \frac{n^2+1}{n^2}$$

$$\textcircled{3} \sum_{n=1}^{+\infty} \frac{2}{\sqrt{n}}$$

$$\textcircled{4} \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$$

$$\textcircled{5} \sum_{n=0}^{+\infty} \frac{1}{5^n}$$

$$\textcircled{6} \sum_{n=2}^{+\infty} \frac{2^n}{3^{n-2}}$$

$$\textcircled{7} \sum_{n \geq 10} \frac{10}{(n+1)^5}$$

$$\textcircled{8} \sum_{n \geq 0} \frac{1}{n!}$$

$$\textcircled{9} \sum_{n \geq 2} \frac{1}{[\ln n]^n}$$

II Calculate the partial derivatives of the following functions:

$$\textcircled{1} f(x,y) = x \cos(xy) \quad \textcircled{2} f(x,y,z) = x + ye^z$$

$$\textcircled{3} f(x,y) = x^2y^3 + 5xy^2 + 2x^3y \quad \textcircled{4} f(x,y) = \cos x \cdot e^y$$