University of Oum El Bouaghi

Faculty of Economic Sciences, Commercial Sciences and Management Sciences

First year, common trunk

Academic year : 2023-2024

Mathematics 1 Module
Second series (Numerical sequences).

Exercise01:
Study the monotonicity and the convergence of the following sequences

$$\mathbf{1)}\, u_n = n^2 + 3, \qquad \mathbf{2)}\; u_n = -4(5)^n\,, \qquad \forall n \in IN.$$

Exercise02:
I) In an arithmetic sequence $(u_n)_{n\in IN}$ whose first term is u_0 and its base is r.

1) Calculate r where : $u_0 = 3$ and $u_{20} - u_{10}=25$.
2) Calculate r, S_8 where : $u_3 = 13$ and $u_7=37$.

II) In an arithmetic sequence $(u_n)_{n\in IN}$ whose first term is $u_0 = 2$ and its base is $r = 5$, calculate the value of n for we will have :

$$\sum_{p=3}^{p=n} u_p = 6456.$$

Exercise03: A student placed an amount of 6000 dinars in the bank with simple interest for several years, meaning that at the end of each year The bank grants an interest of 8% to increase its savings every year by a fixed amount equal to 8% of the initial amount. Assume u_n that represent the amount in year n .

1) Calculate: u_0 , u_1, u_2.

2) Express u_n in terms of n.

3) How many years must we wait for the initial amount to triple.

Exercise04:

Let the sequence (u_n) defined by the general term expression : $u_n = 2^n, \forall\, n \in IN$.

1) Calculate : u_0 , u_1, u_2.

2) Prove that (u_n) is a geometric sequence and determine its basis and first term.

3) Calculate the sum : $u_0 + u_1 + \cdots + u_n$.

4) Study the convergence of $(u_n)_{n\in IN}$.

Exercise05

Let the regressive sequences $(u_n)_{n\in IN}$ and $(v_n)_{n\in IN}$ such that :

$$\begin{cases} v_{n+1} = \frac{u_n+2v_n}{3} \\ u_{n+1} = \frac{2u_n+v_n}{3} \end{cases} \quad , \; v_0 = 2, \; u_0 = 1.$$

And let the sequence $(w_n)_{n\in IN}$ with : $w_n = v_n - u_n$.

1) Prove that $(w_n)_{n\in IN}$ is a geometric sequence.
2) Prove that $(u_n)_{n\in IN}$ and $(v_n)_{n\in IN}$ converge to the same limit.

Exercise06:

A person deposited an amount of 11000 DZD in a bank in 2000 and earned an annual compound interest of 6%. This means that at each end of the year the amount increases with interest of 6% of the previous year's amount. If we consider the deposited amount to be u_0 and consider the number u_n to be the new balance after n years

1) Calculate the amount received in 2001, 2002, 2003.

2) Find a relationship between u_{n+1} and u_n.

3) Deduce the nature of the sequence $(u_n)_{n\in IN}$.

Exercise07:

Let the sequence (u_n) defined by :

$$\begin{cases} u_0 = 2 \\ u_{n+1} = \dfrac{u_n}{3} + 2, \forall n \in IN. \end{cases}$$

1) Calculate: u_0 , u_1, u_2.

2) Prove by regression that : $\forall n \in IN, u_n \leq 3$.

3) Prove that $(u_n)_{n\in IN}$ is increasing.

4) Conclude with justification that $(u_n)_{n\in IN}$ is convergent.

Suggested exercises

First exercise

In the year 2000, the price of one gram of pure gold was estimated at 1000 DZD, noting that the price of the latter increased every year by 20% of the amount it was the previous year.

1) A student bought a ring weighing 4 grams on 01/01/2000. How much will this ring cost on 01/01/2007?

2) This student wanted to sell her ring in 2007 to a jeweler. What is the selling price for this ring, knowing that the jeweler takes a percentage of the profit estimated at 20% of the total amount of the ring**?**

The second exercise

A milk production establishment produces the same amount of milk every day. After two days, the amount of milk produced was 1500 L, and at the end of the fourth day, production was 3000 L.

If milk production forms an arithmetic sequence $(u_n)_{n\in IN}$.

1) Determine its basis and first term.

2) What is the milk production after 25 days?