University of Oum El Bouaghi

Faculty of Economic Sciences, Commercial Sciences and Management Sciences

First year, common trunk

Academic year : 2023-2024

Mathematics 1 Module
Solution of First series (Combinatorial Analysis).

exercise01:

$$\frac{13!}{11!} = 13 \cdot 12 \ , \quad \frac{600!}{598!} = 600 \cdot 599 \ , \qquad \frac{20!}{2! \cdot 3! \cdot 5!} = 10 \cdot 19 \ldots \ldots . \cdot 7 \ ,$$

$$\frac{200!}{2! \cdot 197!} = 100 \cdot 199 \cdot 198 \ , \quad \frac{0!}{12!} = \frac{1}{12} \ , \quad \frac{3! \cdot 15!}{12!} = 6 \cdot 15 \ \cdot 14 \cdot 12$$

$$4! \cdot 3! = 144 \ , \quad 2 \cdot 3! = 12 \ , \quad (4 \cdot 3)! = 12! \ , \quad 5! + 7! = 120 \ \cdot 5040 \ ,$$

$$(5+7)! = 12! \ , \quad 12! - 12! = 0 \ , \qquad 0! \cdot 3! \ = 6$$

Exercise 02:

$$\frac{(n+1)!}{n!} = (n+1) \ , \qquad \frac{(n-1)!}{(n+1)!} = \frac{1}{n(n+1)},$$

$$\frac{n!}{(n-2)!} = n\,(n-1) \ , \qquad \frac{2n!}{(2n-5)!} = 2n\,(n-1)\,(n-2)(n-3)(n-2)$$

Exercise 03:

$$A_5^2 \ = \frac{\mathbf{5!}}{\mathbf{(5-2)!}} = 20 \ , \quad A_{10}^{10} = \frac{10!}{(10-10)!} = 10! \qquad , \quad A_{10}^{0} = \frac{1}{(10-0)!} = \frac{1}{10!},$$

$$A_{50}^{1} \cdot A_{25}^{12} = \frac{50!}{(50-1)!} \cdot \frac{25!}{(25-12)!}$$

$$C_3^2 = \frac{3!}{2!\,(3-2)!} = 3, \quad C_{10}^{5} = \frac{10!}{5!\,(10-5)!} = 252 \quad ,$$

$$C_{10}^{0} + C_5^1 = \frac{10!}{0!\,(10-0)!} + \frac{5!}{1!\,(5-1)!} = 1 + 5 = 6,$$

$$C_5^1 \cdot C_5^2 = \frac{5!}{1!\,(5-1)!} \cdot \frac{5!}{2!\,(5-2)!} = 5 \cdot 10 = 50$$

Exercise 04:

$$\begin{aligned}
C_{n-1}^{p-1} + C_{n-1}^{p} &= \frac{(n-1)!}{(p-1)!\big((n-1)-(p-1)\big)!} + \frac{(n-1)!}{p!\big((n-1)-p\big)!} \\
&= \frac{(n-1)!}{(p-1)!(n-p)!} + \frac{(n-1)!}{p!(n-1-p)!} \\
&= \frac{p(n-1)!}{p(p-1)!(n-p)!} + \frac{(n-p)(n-1)!}{p!(n-p)(n-p-1)!} \\
&= \frac{p(n-1)!}{p!(n-p)!} + \frac{(n-p)(n-1)!}{p!(n-p)!} \\
&= \frac{(n-1)!\big(p+n-p\big)}{p!(n-p)!} \\
&= \frac{n(n-1)!}{p!(n-p)!} \\
&= \frac{n!}{p!(n-p)!} = C_n^p
\end{aligned}$$

$C_6^1 = C_5^1 + C_5^0$ و $C_5^2 = C_4^2 + C_4^1$

exercise 05:

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

$$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

$$\begin{aligned}
(a+b)^2 &= \sum_{p=0}^{2} C_2^p \; a^{2-p}\; b^p \\
&= C_2^0\, a^2 b^0 + C_2^1\, a^{2-1} b^1 + C_2^2\, a^{2-2} b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}$$

$$(2x-3)^5 = \sum_{p=0}^{5} C_p^5 (2x)^{4-p} (-3)^p$$
$$= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x^2 - 243$$

$$(x-y)^7 = \sum_{p=0}^{7} C_7^p x^{7-p} (-y)^p$$
$$= x^7 + 7x^6(-y) + 21x^5(-y)^2 + 35x^4(-y)^3 + 35x^3(-y)^4 + 21x^2(-y)^5 + 7x(-y)^6 + (-y)^7$$

Exercise 06:

1) Using the Newton's Binomial Theorem

$$2^n = (1+1)^n = \sum_{p=1}^{n} C_n^p\, 1^{n-p} 2^p$$
$$= C_n^0 + C_n^1 + C_n^2 + \cdots + C_n^n.$$

2) We have : $96^3 = (100-4)^3$, then we have using the Newton's Binomial.

Exercise07 : a) The first equation is defined for $n \geq 5$, then

$$10C_n^5 = C_n^3 \leftrightarrow 10\frac{n!}{5!\,(n-5)!} = \frac{n!}{3!\,(n-3)!}$$

$$\leftrightarrow \frac{n(n-1)(n-2)(n-3)(n-4)}{2.3!} = \frac{n(n-1)(n-2)}{3!}$$
$$\leftrightarrow (n-3)(n-4) = 2 \leftrightarrow n^2 - 7n + 10 = 0$$

$$s=\{5\},$$

b)

$$\frac{(n+1)!}{n!} = 20 \leftrightarrow n+1 = 20 \leftrightarrow n = 19, \quad s = \{19\},$$

c)

$$A_n^2 = 1 \leftrightarrow \frac{n!}{(n-2)!} = 1 \leftrightarrow n(n-1) = 1 \leftrightarrow n^2 - n - 1 = 0$$