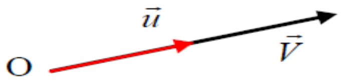
Reminder about vectors

THE UNIT VECTOR : it is a module vector equal to unity (the number one).We can express a vector parallel to the unit vector in the form:

$$\vec{V} = \vec{u}V = V\vec{u}$$



Sum of two Vectors :

We calculate the module of the resulting vector from the **law of cosines** which we will demonstrate later:

$$D = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

Subtracting two vectors $\vec{D} = \vec{V}_2 - \vec{V}_1$

the vector D represents the result of the subtraction between the two vectors

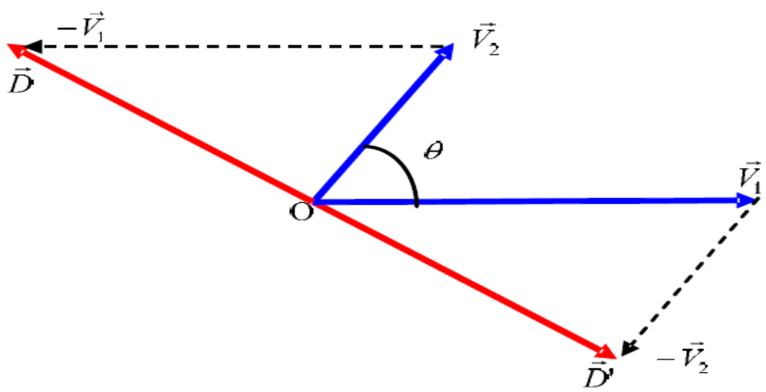
We can write $\vec{D} = \vec{V}_2 - \vec{V}_1$

This equation can also _

: to be written: $\vec{D} = \vec{V}_2 + (-\vec{V}_1)$

The module of vector D

$$\boxed{D = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}}$$



Différence de deux vecteurs

Dot product between two vectors

Let there be two vectors and their scalar product is a product which gives as result a scalar

$$\vec{V}_1 . \vec{V}_2 = \left|\vec{V}_1\right| . \left|\vec{V}_2\right| . \cos\alpha$$

Such that α is the angle between the two vectors

Let the coordinates of the vector (x1,y1,z1) and that of the vector (x2,y2,z2) be
Their scalar product gives

$$\vec{V}_1 . \vec{V}_2 = \left(x_1 \vec{i} + y_1 \vec{j} + z_1 \vec{k}\right)\left(x_2 \vec{i} + y_2 \vec{j} + z_2 \vec{k}\right)$$

$$\vec{V}_1 . \vec{V}_2 = \left(x_1 x_2 + y_1 y_2 + z_1 z_2\right)$$

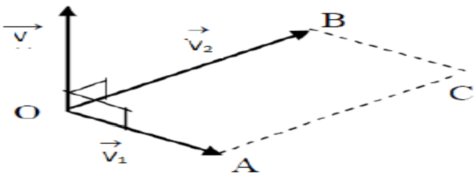
From where

$$\left|\vec{V}_1\right| = \sqrt{\left(x_1^{\,2} + y_1^{\,2} + z_1^{\,2}\right)}$$

Vector product

Let two vectors be $\vec{V_1}$ et $\vec{V_2}$ their cross product is a directed vector

$$\vec{V}_1 \wedge \vec{V}_2 = \vec{V}$$



the direction is perpendicular to the plane formed by the vectors

its standard is

$$\left|\vec{V}_1 \wedge \vec{V}_2\right| = \left|\vec{V}_1\right| . \left|\vec{V}_2\right| . \sin\alpha \qquad \vec{i} \wedge \vec{i} = 0 \; \vec{i} \wedge \vec{j} = \vec{k}$$

$$\vec{V}_1 \wedge \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} \vec{i} - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} \vec{j} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \vec{k} =$$

$$= (y_1 z_2 - z_1 y_2)\vec{i} - (x_1 z_2 - z_1 x_2)\vec{j} + (x_1 y_2 - y_1 x_2)\vec{k}$$

directed work 1

Exercise1:

In a homogeneous orthogonal feature OXYZ we consider the following three rays:

$$\vec{V}_1 = 3\vec{i} - 4\vec{j} + 4\vec{k} \qquad \vec{V}_3 = 5\vec{i} - \vec{j} + 3\vec{k} \qquad \vec{V}_2 = 2\vec{i} + 3\vec{j} - 4\vec{k}$$

1- Calculate their modules.
2- Calculate the components and modules of the vectors:

$$\vec{A} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3 \qquad \vec{B} = 2\vec{V}_1 - \vec{V}_2 + \vec{V}_3$$

3- Determine the unit vector $\vec{U}$ carried by the vector $\vec{C} = \vec{V}_1 + \vec{V}_3$

4- Calculate the scalar and vector product of the vectors $\vec{V}_1 . \vec{V}_3$ and then deduce the angle between them

3- Calculate the products $\vec{V}_2 \wedge \vec{V}_3$.

Exercise2:

In a homogeneous orthogonal feature OXYZ, we consider the two points P (2,-1,3), Q (5,1,-1).

1- Give the components of the ray, $\overrightarrow{PQ}$ then calculate the distance between P and Q.

2- Find $\vec{U}$, the unit radius of the ray $\overrightarrow{OA}$, where $\overrightarrow{PQ} = \overrightarrow{OA}$

Exercise3:

1/ Prove that the area of a parallelogram is $|\vec{A} \wedge \vec{B}|$ where $|\vec{A}|$ and $|\vec{B}|$ are two sides of the parallelogram formed by the two rays.

2/ Prove that these two rays are perpendicular if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

Exercise 4:

Find the sum of the following three rays

$$\vec{V}_1 = 5\vec{i} - 2\vec{j} + 2\vec{k} \qquad \vec{V}_2 = -3\vec{i} + \vec{j} - 7\vec{k} \qquad \vec{V}_3 = 4\vec{i} + 7\vec{j} + 6\vec{k}$$

Calculate the long resultant as well as the angles formed with the axes OX, OY, OZ .