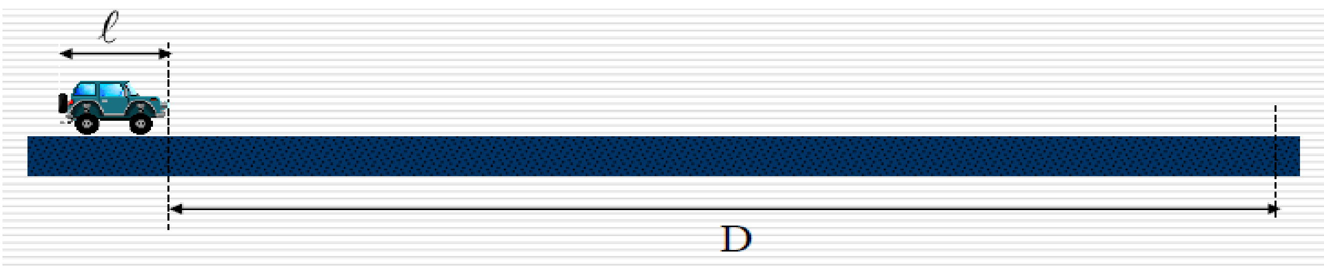
Chapter 1: Kinematics of the material point

I. **Def** . Kinematics is a branch of mechanics which studies the movements of bodies in space as a function of time independently of the causes which cause them.

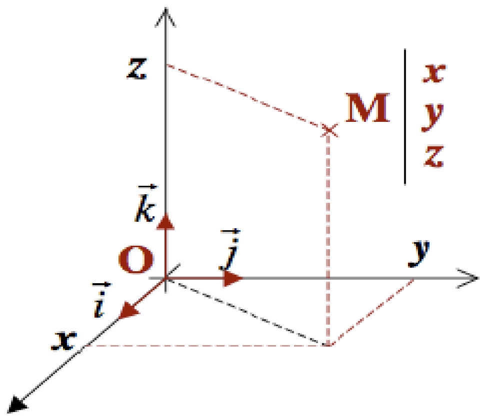
1/ Material point

A material point is an object that is infinitely small compared to the characteristic distances of movement to be considered punctual.



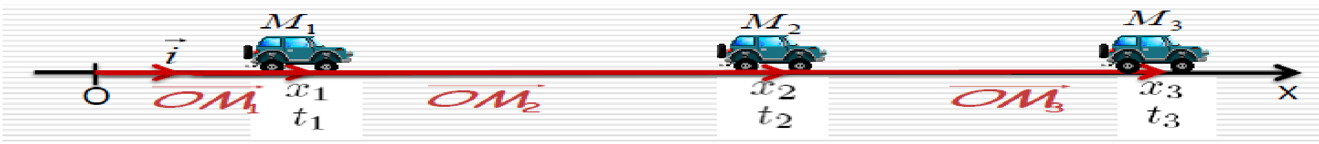
- ✓ A point M in space is identified by its coordinates x, y and z such that:

$$\overrightarrow{OM} = \vec{r} = x.\vec{i} + y.\vec{j} + z.\vec{k}$$



2/ Position vector

It is the vector $\overrightarrow{OM} = x(t)\vec{i}$ which designates the distance which separates the mobile M from the point O taken as the origin.



x(t) is called the time equation of motion.

a/ Cartesian equation of the trajectory:

The trajectory is the set of positions occupied by the mobile during its movement for successive moments.

We obtain the equation of the trajectory by eliminating the time between the two time equations x(t) and y(t).

ex : **The** time equations of the movement of a material point drawn in space are $x = 2t$; $y = 0$; $z = -5t^2 + 4t$ (all units are in the international system).

1/ Find the Cartesian equation of the trajectory, what is its form?

2/ Write the expression for the position vector at time $t = 2$ s.

Answer:

1/

$$x = 2t \Rightarrow t = \frac{x}{2}$$

$$\boxed{z = -1.25.x^2 + 2.x}$$ Equation of a parable

2/ Expression position vector :

$$\overrightarrow{OM} = x.\vec{i} + y\vec{j} + z.\vec{k}$$

$$\overrightarrow{OM} = (2t).\vec{i} + (-5t^2 + 4t).\vec{k} \Rightarrow \overrightarrow{OM}_{(t=2)} = 4\vec{i} - 12\vec{k}$$

$$\boxed{\overrightarrow{OM}_{(t=2)} = 4\vec{i} - 12\vec{k}}$$

b/ Displacement vector :

The displacement vector is the distance traveled between two instants.

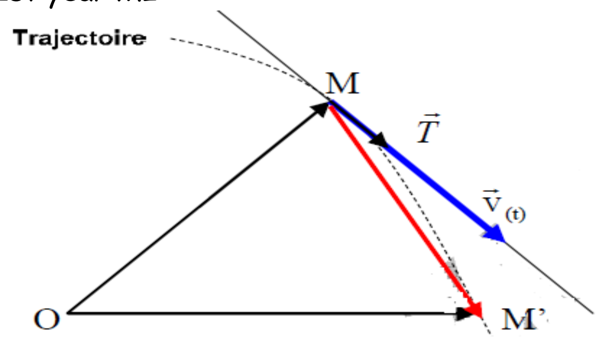
$$\overrightarrow{M_1M_2} = \Delta\overrightarrow{OM} = \overrightarrow{OM_2} - \overrightarrow{OM_1} = (x_2 - x_1)\vec{i}$$

3/ THE SPEED VECTOR:

Speed is considered to be the distance traveled per unit of time.

a/ Average speed vector

Let's look at the figure above: between time t where the mobile occupies position M , and time t' where the mobile occupies position M', the average speed vector is defined as being the expression:



$$\vec{v}_{moy} = \frac{\overrightarrow{MM'}}{t'-t} \quad ; \quad v_{moy} = \frac{\left|\overrightarrow{MM'}\right|}{\Delta t}$$

b/ Instantaneous speed vector:

The instantaneous velocity vector, i.e. at time t , is the derivative of the position vector relative to time:

$$\vec{v}_t = \lim_{t \to t'} \frac{\overrightarrow{OM'} - \overrightarrow{OM}}{t - t'} = \lim_{t' \to t} \frac{\Delta \overrightarrow{OM}}{\Delta t} = \frac{d\overrightarrow{OM}}{dt}$$

$$\vec{v}_t = \frac{d\overrightarrow{OM}}{dt}$$

Modulus of the instantaneous velocity vector is:

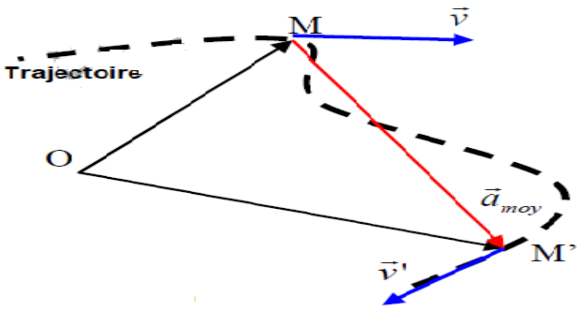
$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

4/ THE ACCELERATION VECTOR :

We consider acceleration to be the change in speed per unit of time.

a/ Average acceleration vector:

By considering two different instants t and t' corresponding to the position vectors $\overrightarrow{OM}$ and $\overrightarrow{OM'}$ and the instantaneous velocity vectors $\vec{V}$ and $\vec{V'}$ (figure), the average acceleration vector is defined by the following expression:



$$\vec{a}_{moy} = \frac{\vec{v}' - \vec{v}}{t' - t} = \frac{\Delta \vec{v}}{\Delta t}; \qquad a_{moy} = \frac{\left|\Delta \vec{v}\right|}{\Delta t}$$

b/ Instantaneous acceleration vector

The instantaneous acceleration vector of a movement is defined as the derived from the instantaneous velocity vector with respect to time.

$$\vec{a} = \lim_{t'\to t} \frac{\vec{v}' - \vec{v}}{t' - t} = \lim_{t'\to t} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{\mathrm{d}^2\overrightarrow{OM}}{dt^2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\overrightarrow{OM}}{dt^2}$$

Modulus of the instantaneous acceleration vector is:

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

- ✓ We can now write in summary the expressions of the position, velocity and acceleration vectors in Cartesian coordinates, with the conventions of Newton and Leibnitz:

$$\overrightarrow{OM} = \vec{r} = x.\vec{i} + y.\vec{j} + z.\vec{k} \Rightarrow \vec{v} = \dot{x}.\vec{i} + \dot{y}.\vec{j} + \dot{z}.\vec{k} \Rightarrow \vec{a} = \ddot{x}.\vec{i} + \ddot{y}.\vec{j} + \ddot{z}.\vec{k}$$

$$\vec{v} = \frac{dx}{dt}.\vec{i} + \frac{dy}{dt}.\vec{j} + \frac{dz}{dt}.\vec{k} \Rightarrow \boxed{\vec{a} = \frac{d^2x}{dt^2}.\vec{i} + \frac{d^2y}{dt^2}.\vec{j} + \frac{d^2z}{dt^2}.\vec{k}}$$

CONCLUSION : In a Cartesian frame of reference the position, speed and acceleration vectors are:

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_R \rightarrow \vec{v} \begin{pmatrix} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \end{pmatrix}_R \rightarrow \vec{a} \begin{pmatrix} \ddot{x} = \dot{v}_x = a_x \\ \ddot{y} = \dot{v}_y = a_y \\ \ddot{z} = \dot{v}_z = a_z \end{pmatrix}_R$$

$$\overrightarrow{OM} = \vec{r} = x.\vec{i} + y.\vec{j} + z.\vec{k} \rightarrow \vec{v} = v_x.\vec{i} + v_y.\vec{j} + v_z.\vec{k} \rightarrow \vec{a} = a_x.\vec{i} + a_y.\vec{j} + a_z.\vec{k}$$

Ex :

Let the position vector be $\overrightarrow{OM}\begin{pmatrix} x = 2t^2 \\ y = 4t - 5 \\ z = t^3 \end{pmatrix}$. Deduce the velocity vector and the instantaneous acceleration vector, then calculate the modulus of each of them.

Answer :

We derive the expression of the position vector twice in a row to obtain the requested vectors and then we deduce their modules:

$$\vec{V} = 4t.\vec{\imath} + 4\vec{\jmath} + 3t^2\vec{k} \rightarrow \vec{\gamma} = 4\vec{\imath} + 0\vec{\jmath} + 6t\vec{k}$$

$$V = \sqrt{16t^2 + 16 + 9t^4}, \qquad \gamma = \sqrt{16 + 36t^2}$$

II. / Rectilinear movement:

1/ Uniform rectilinear movement

Def. A movement is said to be uniform rectilinear if the trajectory is a straight line and the speed is constant.

Time equation : we choose the *OX axis* as a rectilinear reference and we set the initial condition *t=0; x=x₀*.

Starting from the definition above, and thanks to an integration we can express the abscissa *x* as a function of time:

$$v = \dot{x} = \frac{dx}{dt} = v_0 \Rightarrow dx = v_0.dt$$

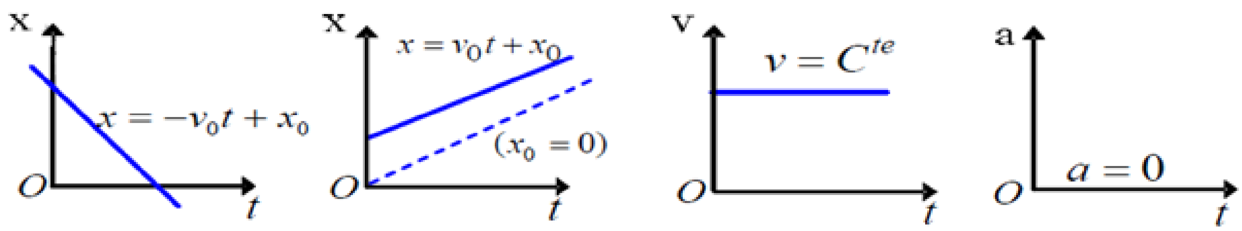
$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v dt = v\int_{t_0}^{t} dt$$

$$x - x_0 = v(t - t_0)$$

$$\Rightarrow x = v(t - t_0) + x_0$$

-Movement diagrams :

Uniform rectilinear motion diagrams are the graphical representation of acceleration, velocity, and displacement as a function of time.



2/ UNIFORMLY VARIED RECTILINEAR MOVEMENT:

- **Definition** : The movement of a material point is rectilinear uniformly varied if its trajectory is a straight line and its acceleration is constant.

- **Algebraic speed** : Considering the initial conditions $t = t_0 = 0$; $v = v_0$ (initial speed), and starting from the previous definitions, and integrating we can write:

$$a = \frac{dv}{dt} \Rightarrow dv = adt$$

$$\Rightarrow v = a(t - t_0) + v_0$$

$$\Rightarrow v = at + v_0$$

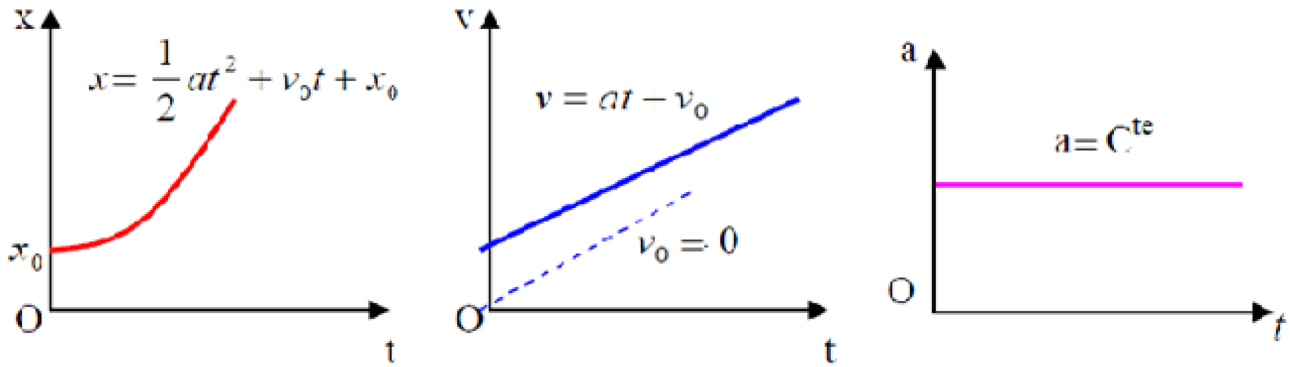
$$\int_{x_0}^{x} dx = \int_{t_0}^{t} vdt = \int_{t_0}^{t} (a(t - t_0) + v_0)dt$$

$$x - x_0 = \frac{1}{2} a(t^2 - t_0^2) - at_0(t - t_0) + v_0(t - t_0)$$

$$\boxed{x = \frac{1}{2} a(t)^2 + v_0 t + x_0}$$

-Movement diagrams :

We see in the figure the diagrams of uniformly varied rectilinear motion relating to acceleration, speed and displacement.



3/ Rectilinear movement with variable acceleration

The movement of a material point is said to be rectilinear with variable acceleration if its trajectory is a straight line and its acceleration is a function of time

$(\gamma = f(t))$.

Example : a point body moves along a straight line with acceleration $\gamma = 4 - t^2$

Find the expressions for speed and displacement as a function of time considering the following conditions: $t = 3s$; $v = 2ms^{-1}$; $x = 9m$

Answer : To obtain the literal expression of the speed we must integrate the acceleration equation:

$$v = \int_0^t \gamma dt + v_0 \Rightarrow v = v_0 + \int_0^t (4 - t^2)\, dt \Rightarrow v = 4t - \frac{1}{3}t^3 + v_0$$

Integrating again to obtain the literal expression of the displacement:

$$x = x_0 + \int_0^t v dt \Rightarrow x = -\frac{1}{12}t^4 + 2t^2 + v_0 t + x_0$$

We still need to determine the initial abscissa and velocity of the body. According to the data, we replace the time in the expressions obtained previously by: $t = 3s$to find the initial abscissa and speed:$t = 3s \Rightarrow x_0 = \frac{3}{4}m;\quad v_0 = -1ms^{-1}$

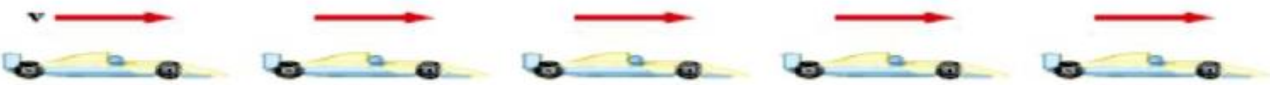
Ultimately, the expressions for velocity and displacement are:

$x = 2t^2 - \frac{1}{12}t^4 - t + \frac{3}{4}$ And $v = 4t - \frac{1}{3}t^3 - 1$

- ✓ It is the uniformly varied movement. This movement can be accelerated or delayed. In the first case the product of the speed and the acceleration must be positive, in the second case the same product must be negative

Uniform rectilinear movement:

$$\vec{v} = constant \quad \vec{a} = \vec{0}$$

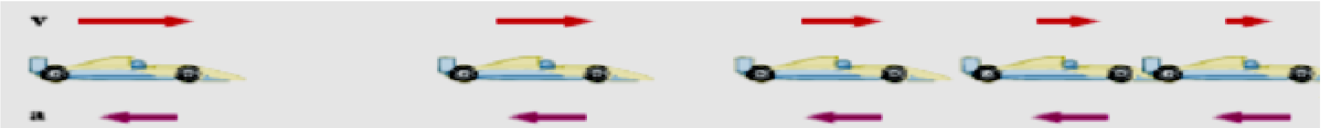


Uniformly accelerated rectilinear motion:

$$\vec{a} = constant \quad \vec{a}.\vec{v} \rangle 0$$

Rectilinear movement uniformly delayed or decelerated:

$\vec{a} = constant$ $\vec{a}.\vec{v}\langle 0$



Applications:

Exercise 1

A material point moves in a straight line following the following time equation:

$$X(t) = -6t^2 + 16t$$

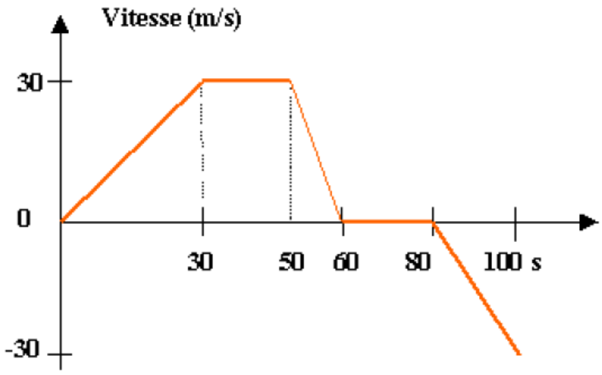
- What is the position of this body$t = 1s$
- At what time t,does it pass through position O (origin).
- What is the average speed in the time interval between $0s$and $2s$.
- Give the expression for the instantaneous speed, deduce its value from$t = 0s$
- What is the average acceleration in the time interval between $0s$and $2s$.
- Give the expression for the instantaneous acceleration.

Solution exercise 1

- body position at $t = 1s$: $x(1) = 10$
- $x = 0 \Rightarrow 6t^2 + 16t = 0$it passes through the origin to $t = 0s$and$t = \frac{8}{3} = 2.7s$
- $v_{moy} = \frac{x(t=2)-x(t=0)}{2-0} = 4m/s$
- $\vec{v}(t) = \frac{dx(t)}{dt}\vec{\imath} = 16 - 12t; v(0) = 16m/s$
- $\gamma_{moy} = \frac{v(t=2)-v(t=0)}{2-0} = -12m/s^2$
- $\gamma = \frac{dv}{dt} = -12$

Exercise 2:

A vehicle travels $x_0 = 0$on a straight path. Its speed is characterized by the following diagram.



1. Indicate over the 5 time intervals: the algebraic value of the acceleration and the displacement
2. Determine at the end of the movement at t=100s: the final position x and the path traveled in absolute value.

Solution ex 2

1. **the algebraic value of the acceleration and the displacement over the 5 time intervals:**

1) $0 < t < 30$ s, $a = \frac{\Delta v}{\Delta t} = 1$ m/s^2, $v = at + v_0$

   A t=0 s, v=0 m/s $\Rightarrow v_0 = 0$. Donc : $v = t$. Le mouvement est uniformiment accéléré.

2) $30 < t < 50$ s, $a = 0$ m/s^2, $v = 30$ m/s. Le mouvement est uniforme.

3) $50 < t < 60$ s, $a = \frac{\Delta v}{\Delta t} = -3m/s^2$, $v = at + v_0$

   A t=0 s, v = 30 m/s $\Rightarrow v_0 = 30$. Donc : $v = -3t + 30$. Le mouvement est uniformiment retardé.

4) $60 < t < 80$ s, a=0 m/s^2, $v = 0$ m/s. Le mobile est au repos.

5) $80 < t < 100$ s, $a = \frac{\Delta v}{\Delta t} = \frac{-3}{2}$ m/s^2, $v = at + v_0$

   A t=0 s, v=0 m/s $\Rightarrow v_0 = 0$. Donc : $v = \frac{-3}{2}t$. Le mouvement est uniformiment accéléré.

$0<t<30s$	$30<t<50s$	$50<t<60s$	$60<t<80$	$80<t<100$
$a_1=1m/s^2$	$a_2=0$	$a_3=-3m/s^2$	$a_4=0$	$a_5=-1,5m/s^2$
$\Delta x_1=450m$	$\Delta x_2=600m$	$\Delta x_3=150m$	$\Delta x_4=0$	$\Delta x_5=-300m$

2. **The final *x position***

abscisse finale : $x=x_0+\sum \Delta x_i = 900m$; *chemin parcouru :* $l= \sum |\Delta x_i| = 1500m$

III. Study of movements in different systems (polar, cylindrical and spherical)

1/ Polar coordinates

It is a coordinate system used to locate the position of a point M in two dimensions (plane motion). The position of point M is identified by its polar coordinates (r, φ).

r:polar ray

φ:polar angle

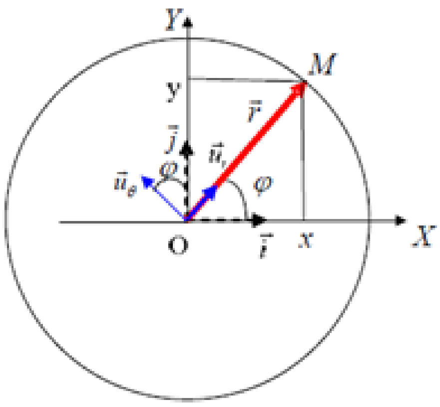


Figure: polar coordinates

So we have : $r = |\overrightarrow{OM}|$; $\quad 0 < r < +\infty$

$\varphi = (\overrightarrow{OM}, \vec{\imath})$; $\quad 0 < \varphi < 2\pi$

Using the diagram in the figure above we can find the relationships between Cartesian coordinates and polar coordinates:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

Or vice versa : $\begin{cases} r = \sqrt{x^2 + y^2} \\ \varphi = arctan\dfrac{y}{x} \end{cases}$

We define the base $(\overrightarrow{u_r}, \overrightarrow{u_\varphi})$associated with the polar coordinates:

$$\begin{cases} \overrightarrow{u_r} = \cos\varphi\,\vec{\imath} + \sin\varphi\,\vec{\jmath} \\ \overrightarrow{u_\varphi} = -\sin\varphi\,\vec{\imath} + \cos\varphi\,\vec{\jmath} \end{cases}$$

- The position vector:$\overrightarrow{OM} = r\overrightarrow{u_r}$
- The velocity vector in polar coordinates:

To obtain the expression of the velocity vector in polar coordinates we derive the position vector in polar coordinates:

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d(r\overrightarrow{u_r})}{dt} = \frac{d(r)}{dt}\overrightarrow{u_r} + r\frac{d(\overrightarrow{u_r})}{dt} \Rightarrow \vec{V} = \dot{r}\overrightarrow{u_r} + r\dot{\varphi}\overrightarrow{u_\varphi}$$

<u>Noticed:</u>

The vector $\overrightarrow{u_r}$being mobile and implicitly depends on t , through its dependence on the angleφ

$$\frac{d(\overrightarrow{u_r})}{dt} = \frac{d(\overrightarrow{u_r})}{dt}\frac{d\varphi}{d\varphi} = \frac{d(\overrightarrow{u_r})}{d\varphi}\frac{d\varphi}{dt} = \dot{\varphi}\overrightarrow{u_\varphi}$$

$$\frac{d(\overrightarrow{u_r})}{d\varphi} = \{\overrightarrow{u_\varphi} = -\sin\varphi\,\vec{\imath} + \cos\varphi\,\vec{\jmath}$$

- The acceleration vector in polar coordinates:

$$\vec{\gamma} = (\ddot{r} - r\dot{\varphi}^2)\vec{u}_r + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\vec{u}_\varphi$$

Example :

Let $M(\sqrt{3};-1)$, determine the polar coordinates of M.

Answer: $r = \sqrt{x^2+y^2} = \sqrt{3+1} = 2$and $\begin{cases}\cos\varphi = \frac{x}{r} = \frac{\sqrt{3}}{2} \\ \sin\varphi = \frac{y}{r} = -\frac{1}{2}\end{cases} \Rightarrow \varphi = -\frac{\pi}{6}$thereforeM $\left(2;-\frac{\pi}{6}\right)$

2/ Cylindrical coordinates

If the trajectory is spatial, where $r$and ozplay a particular role in determining the position of the mobile, it is preferable to use cylindrical coordinates (r,φ,z) :

r:polar ray

φ:polar angle

z:altitude

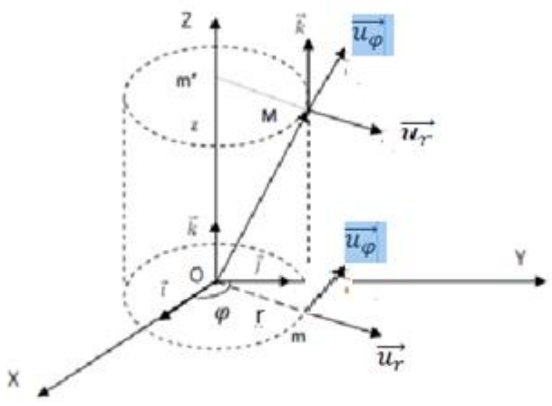


Figure: cylindrical coordinates

We therefore have according to the figure:

$$\begin{cases} r = |\overrightarrow{Om}|, & 0 \leq r \leq +\infty \\ \varphi = (\overrightarrow{Om}, \vec{\imath}); & 0 \leq \varphi \leq 2\pi \\ z = |\overrightarrow{Om'}|; & -\infty < z < +\infty \end{cases}$$

- Rules for transitioning from Cartesian coordinates to cylindrical coordinates:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{cases}$$

Or vice versa

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \varphi = arctan\dfrac{y}{x} \\ z = z \end{cases}$$

We associate the orthonormal base $(\overrightarrow{u_r}, \overrightarrow{u_\varphi}, \vec{k})$with the cylindrical coordinates, this base is connected to the base of the Cartesian coordinates by the relations:

$$\begin{cases} \overrightarrow{u_r} = \cos\varphi\ \vec{\imath} + \sin\varphi\,\vec{\jmath} \\ \overrightarrow{u_\varphi} = -\sin\varphi\,\vec{\imath} + \cos\varphi\,\vec{\jmath} \\ \vec{k} = \vec{k} \end{cases}$$

- **Position vector**

  In this basis the position vector is written as follows:

$$\overrightarrow{OM} = \overrightarrow{Om} + \overrightarrow{mM} = r\overrightarrow{u_r} + z\vec{k}$$

- **Velocity vector**

To obtain the expression of the velocity vector in cylindrical coordinates we derive the position vector in cylindrical coordinates:

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d(r\overrightarrow{u_r} + z\vec{k})}{dt} = \frac{dr}{dt}\vec{u}_r + r\frac{d\vec{u}_r}{dt} + \frac{dz}{dt}\vec{k} + z\frac{d\vec{k}}{dt}$$

Knowing that $\vec{k}$is a fixed vector its derivative is zero $\frac{d\vec{k}}{dt} = 0$. The vector $\vec{u}_r$being mobile, its derivative is not zero in general. Indeed, $\vec{u}_r$depends implicitly on t , through its dependence on the angle φ. So :

$$\frac{d\vec{u}_r}{dt} = \frac{d\vec{u}_r}{d\varphi}\frac{d\varphi}{dt}$$

Using the expression of the vector $\vec{u}_r$in the base $(\vec{\imath}, \vec{\jmath}, \vec{k})$we obtain:

$$\frac{d\vec{u}_r}{dt} = \frac{d\vec{u}_r}{d\varphi}\frac{d\varphi}{dt} = \frac{d\varphi}{dt}\vec{u}_\varphi = \dot{\varphi}\vec{u}_\varphi$$

We then obtain for the speed vector:

$$\vec{V} = \frac{dr}{dt}\vec{u}_r + r\frac{d\varphi}{dt}\vec{u}_\varphi + \frac{dz}{dt}\vec{k} = \dot{r}\vec{u}_r + r\dot{\varphi}\vec{u}_\varphi + \dot{z}\vec{k}$$

- Acceleration vector in cylindrical coordinates:

$$\vec{\gamma} = (\ddot{r} - r\dot{\varphi}^2)\vec{u}_r + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\vec{u}_\varphi + \ddot{z}\vec{k}$$

3/ Spherical coordinates

When the point O and the distance separating M from O play a characteristic role, the use of spherical coordinates $(r, \theta, \varphi)$is best suited.

r: polar ray

θ : coaltitude

φ : azimuth

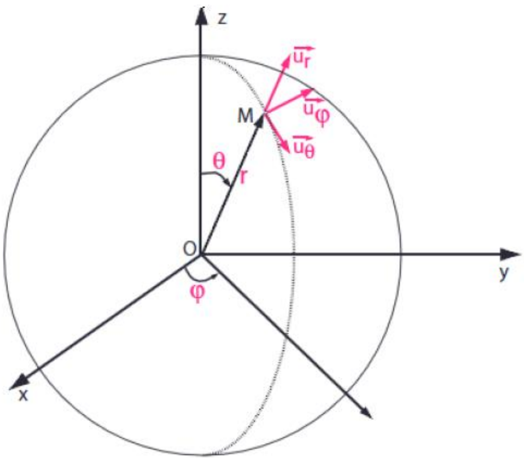


Figure: spherical coordinates

We therefore have according to the figure:

$$\begin{cases} r = \left|\overrightarrow{oM}\right| \quad ; \quad 0 \le r < +\infty \\ \varphi = \left(\widehat{\overrightarrow{om}, \vec{\imath}}\right) \quad ; \quad 0 \le \varphi < 2\pi \\ \theta = \left(\widehat{\overrightarrow{OM}, \vec{k}}\right) ; \quad 0 \le \theta \le \pi \end{cases}$$

Rules for transitioning from Cartesian coordinates to spherical coordinates:

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

Or vice versa :

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \arctan\dfrac{\sqrt{x^2+y^2}}{z} \\ \varphi = \dfrac{y}{x} \end{cases}$$

We associate the orthonormal base $(\overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{u_\varphi})$with the spherical coordinates, this base is connected to the base of the Cartesian coordinates by the relations:

$$\begin{cases} \vec{u}_r = \sin\theta\cos\varphi\,\vec{\imath} + \sin\theta\sin\varphi\,\vec{\jmath} + \cos\theta\,\vec{k} \\ \vec{u}_\theta = \cos\theta\cos\varphi\,\vec{\imath} + \cos\theta\sin\varphi\,\vec{\jmath} - \sin\theta\,\vec{k} \\ \vec{u}_\varphi = -\sin\varphi\,\vec{\imath} + \cos\varphi\,\vec{\jmath} \end{cases}$$

- Position vector: in this basis, the position vector is written as follows:

$$\overrightarrow{OM} = r\vec{u}_r$$

- **Velocity vector in spherical coordinates**

  The position vector in spherical coordinates depends on the vector $\vec{u}_r$. The latter depends on the angles $\theta$and φ, therefore, its derivative with respect to time is given by:

$$\frac{d\vec{u}_r}{dt} = \frac{d\vec{u}_r}{d\theta}\frac{d\theta}{dt} + \frac{d\vec{u}_r}{d\varphi}\frac{d\varphi}{dt}$$

Using the expressions of the vectors $(\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi)$as a function of the vectors $(\vec{\imath}, \vec{\jmath}, \vec{k})$given previously, we show that:

$\frac{d\vec{u}_r}{d\theta} = \vec{u}_\theta$And$\frac{d\vec{u}_r}{d\varphi} = \sin\theta\,\vec{u}_\varphi$

So$\frac{d\vec{u}_r}{dt} = \frac{d\theta}{dt}\vec{u}_\theta + \sin\theta\,\frac{d\varphi}{dt}\,\vec{u}_\varphi$

The velocity vector is obtained by deriving the position vector:

$$\vec{V}=\frac{d\overrightarrow{OM}}{dt}=\frac{dr}{dt}\vec{u}_r+r\frac{d\vec{u}_r}{dt}=\frac{dr}{dt}\vec{u}_r+r\frac{d\theta}{dt}\vec{u}_\theta+r\sin\theta\,\frac{d\varphi}{dt}\,\vec{u}_\varphi$$

Or :$\vec{V}=\dot{r}\vec{u}_r+r\dot{\theta}\vec{u}_\theta+r\dot{\varphi}\sin\theta\vec{u}_\varphi$

- Acceleration vector

$$\vec{\gamma}=\left(\ddot{r}-r\dot{\theta}^2-r\sin^2\theta\dot{\varphi}^2\right)\vec{u}_r+\left(2\dot{r}\dot{\theta}+r\ddot{\theta}-r\sin\theta\cos\theta\dot{\theta}^2\right)\vec{u}_\theta$$
$$+\left(2\dot{r}\sin\theta\dot{\varphi}+r\sin\theta\ddot{\varphi}+2r\cos\theta\dot{\theta}\dot{\varphi}\right)\vec{u}_\varphi$$

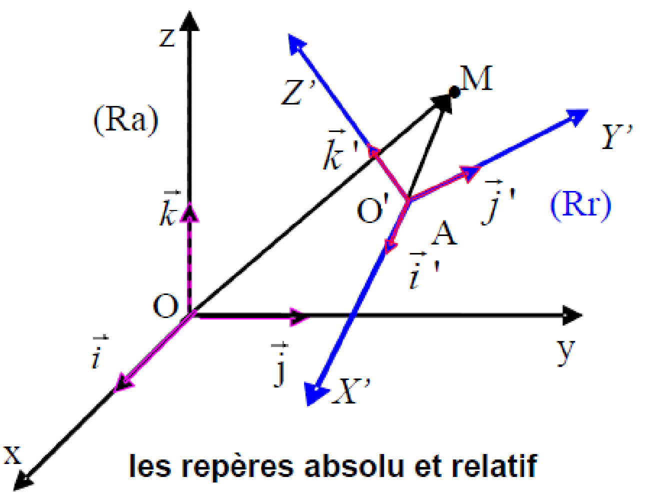
IV. RELATIVE MOVEMENT:

The movement of a material point can be divided into two distinct movements

- A movement relative to a fixed reference point which we will call Absolute reference point
- A movement relative to a mobile reference which we will call relative reference

a/ Absolute and relative quantities

Let a material point M be in motion relative to a relative reference frame Rr (o',x',y',z'), itself in motion relative to an absolute reference frame Ra (O,x,y,z).



les repères absolu et relatif

b/ The position:

The position of M in Ra is the absolute position and its position in Rr is its relative position

$$\overrightarrow{OM}=x\vec{i}+y\vec{j}+z\vec{k}$$

$$\overrightarrow{O'M}=x'\vec{i}'+y'\vec{j}'+z'\vec{k}'$$

$$\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M}$$

c/ Speed:

Absolute speed is the speed of M relative to Ra

$$\vec{V}_a = \frac{d\overrightarrow{OM}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

This speed can be calculated in another way:

$$\frac{d\overrightarrow{OM}}{dt} = \frac{d\overrightarrow{OO'}}{dt} + \frac{d\overrightarrow{O'M}}{dt}$$

$$\frac{d\overrightarrow{OM}}{dt} = \frac{d\overrightarrow{OO'}}{dt} + x'\frac{d\vec{i'}}{dt} + y'\frac{d\vec{j'}}{dt} + z'\frac{d\vec{k'}}{dt} + \frac{dx'}{dt}\vec{i'} + \frac{dy'}{dt}\vec{j'} + \frac{dz'}{dt}\vec{k'}$$

We pose:

$$\vec{V}_e = \frac{d\overrightarrow{OO'}}{dt} + x'\frac{d\vec{i'}}{dt} + y'\frac{d\vec{j'}}{dt} + z'\frac{d\vec{k'}}{dt}$$

$$\vec{V}r = \frac{dx'}{dt}\vec{i'} + \frac{dy'}{dt}\vec{j'} + \frac{dz'}{dt}\vec{k'}$$

From where

$$\boxed{\vec{V}_a = \vec{V}_e + \vec{V}_r}$$

$\overrightarrow{V_r}$: it is the relative speed, that is to say the speed of the mobile M relative to the reference Rr.

$\vec{V}a$: represents the training speed, that is to say the speed of the Rr reference mark relative to the Ra reference mark.

d/ Acceleration

The absolute acceleration is the acceleration of the point M in the frame R:

$$\vec{a}_a = \frac{d^2\overrightarrow{OM}}{dt^2} = \frac{d\vec{v}_a}{dt} = \left[\frac{d^2\overrightarrow{OA}}{dt^2} + x'\frac{d^2\vec{i}\,'}{dt^2} + y'\frac{d^2\vec{j}\,'}{dt^2} + z'\frac{d^2\vec{k}\,'}{dt^2}\right] \rightarrow \vec{a}_e$$

$$+\left[\vec{i}\,'\frac{d^2x'}{dt^2} + \vec{j}\,'\frac{d^2y'}{dt^2} + \vec{k}\,'\frac{d^2z'}{dt^2}\right] \rightarrow \vec{a}_r$$

$$+2\left[\frac{dx'.d\vec{i}\,'}{dt^2} + \frac{dy'.d\vec{j}\,'}{dt^2} + \frac{dz'.d\vec{k}\,'}{dt^2}\right] \rightarrow \vec{a}_C$$

$\vec{a}_a$: absolute acceleration: it is the acceleration of *M* relative to the reference frame (*Ra*) .

$\vec{a}_r$: relative acceleration: this is the acceleration of *M* relative to the reference frame (*Rr*) .

$\vec{a}_e$: training acceleration: it is the acceleration of the reference (*Rr*) relative to

at the mark (*Ra*) .

$\vec{a}_C$: Coriolis acceleration: it is a complementary acceleration called **Coriolis acceleration**.