L'arbi Ben M'hidi University

Faculty: Exact sciences , natural and life sciences
Department: MI
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Module Algebra 1

Serie 2

Exercise 1 :
Let E be a set and A, B and C three parts of E, prove that :
1. $C_E(A \cap B) = C_E A \cup C_E B$.
2. $C_E(A \cup B) = C_E A \cap C_E B$.
3. $A/(B \cap C) = A/B \cup A/C$.
4. $A \subset B \Leftrightarrow A \cup B = B$
5. $A \cup B = A \cap C \Leftrightarrow B \subset A \subset C$
Exercise 2 :
Let's consider the applications f and g defined by

$$f \quad : \quad [0,1] \to [0,2] \;, \quad g : [-1,1] \to [0,2]$$
$$x \quad \longmapsto \quad 2-x \qquad\qquad x \longmapsto x^2+1$$

(1) Determine $f\left(\dfrac{1}{2}\right), f^{-1}(\{0\}), g([-1,1]), g^{-1}([0,2])$
(2) Is the application f bijective? justify.
(3) Is the application g bijective? justify.
Exercise 3 :
Let's consider the applications $f : A \to B$, $g : B \to C$ and $h : C \to D$. Prove that
(1) $g \circ f$ injective$\Rightarrow f$ injective
(2) $g \circ f$ surjective$\Rightarrow g$ surjective
(3) $g \circ f$ and $h \circ g$ are bijectives$\Leftrightarrow (f,\; g$ and h are bijectives$)$.

Pr. Rezzag.S