Université Larbi Ben M'Hidi d'OEB
Faculté des Sciences et des Sciences Appliquées
2ième année Licence Génie Mécanique, Génie civile et Hydraulique

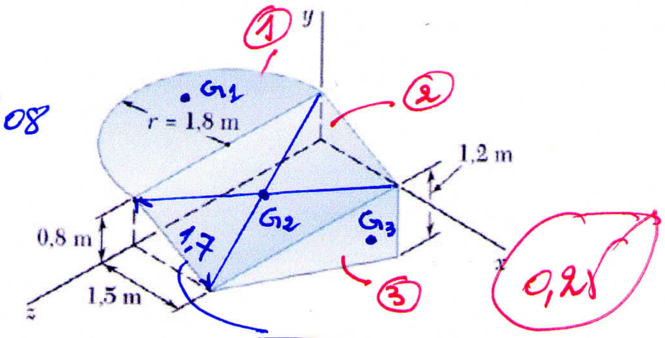
le 25/01/2022

Durée 1H30

# Corrigé Type : Examen de Mécanique Rationnelle

**Exercice 1** : (7 pts) Localisez le centre de gravité de la surface composée suivante :

$$\vec{OG_1} = \begin{cases} -\frac{4r}{3\pi} = -0{,}76 \quad (0{,}5) \\ 0{,}8 \quad (0{,}5) \\ 1{,}8 \quad (0{,}5) \end{cases} \qquad S_1 = \frac{\pi r^2}{2} = 5{,}08 \quad (0{,}5)$$




$$\sqrt{0{,}8^2 + 1{,}5^2} = 1{,}7$$

$$\vec{OG_2} = \begin{cases} \frac{1{,}5}{2} = 0{,}75 \quad (0{,}5) \\ 0{,}8/2 = 0{,}4 \quad (0{,}5) \\ 1{,}8 \quad (0{,}5) \end{cases} \qquad S_2 = (2\times1{,}8)\times1{,}7 \qquad S_2 = 6{,}12 \quad (0{,}5)$$

$$\vec{OG_3} = \begin{cases} 1{,}5 \quad (0{,}5) \\ -\frac{1{,}2}{3} = -0{,}4 \quad (0{,}5) \\ \frac{3{,}6}{3} = 1{,}2 \quad (0{,}5) \end{cases} \qquad S_3 = \frac{2{,}6\times1{,}2}{2} = 1{,}56 \quad (0{,}5)$$

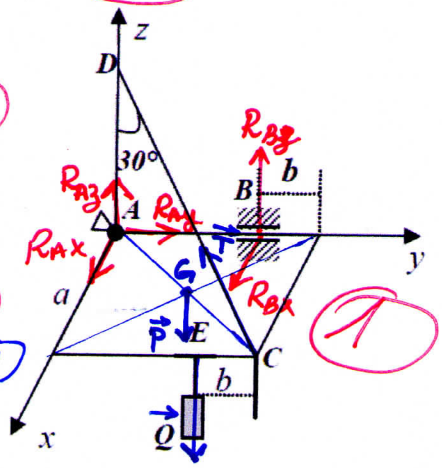
$$x_G = \frac{\sum x_{Gi} S_i}{\sum S_i} \qquad y_G = \frac{\sum y_{Gi} S_i}{\sum S_i} \qquad z_G = \frac{\sum z_{Gi} S_i}{\sum S_i}$$

$$x_G = \frac{x_{G1} S_1 + x_{G2} S_2 + x_{G3} S_3}{S_1 + S_2 + S_3}$$

$$x_G = \frac{-0{,}76 \cdot 5{,}02 + 0{,}75 \cdot 6{,}12 + 1{,}5 \cdot 1{,}56}{5{,}08 + 6{,}12 + 1{,}56} = 0{,}24\ m \quad (0{,}25)$$

$$y_G = 0{,}46\ m \quad (0{,}25) \ ; \quad z_G = 1{,}72\ m. \quad (0{,}25)$$

**Exercice 2** : (9 pts)




$$\vec{R_A}\begin{pmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{pmatrix} (0{,}5) \ ; \ \vec{R_B}\begin{pmatrix} R_{Bx} \\ 0 \\ R_{Bz} \end{pmatrix} (0{,}5) \ ; \ \vec{P}\begin{pmatrix} 0 \\ 0 \\ -P \end{pmatrix} (0{,}5) \ ; \ \vec{Q}\begin{pmatrix} 0 \\ 0 \\ -Q \end{pmatrix} (0{,}5)$$

$$\sum \vec{F_i} = \vec{0} \Rightarrow \vec{R_A} + \vec{R_B} + \vec{T} + \vec{P} + \vec{Q} = \vec{0} \quad (0{,}5) \quad — (1)$$

$$\vec{T} = \begin{pmatrix} -T\cos60°\cos45° \\ -T\cos60°\sin45° \\ T\sin60 \end{pmatrix} = T\begin{pmatrix} -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (0{,}5)$$

$$(1) \Leftrightarrow \begin{cases} R_{Ax} + R_{Bx} - T\frac{\sqrt{2}}{4} = 0 \\ R_{Ay} - T\frac{\sqrt{2}}{4} = 0 \\ R_{Az} + R_{Bz} + T\sin60° - 3P = 0 \end{cases} \quad (0{,}75)$$

* $\sum \vec{M}_A(\vec{F}_i) = \vec{0}$

$\vec{AB} \wedge \vec{R_B} + \vec{AC} \wedge \vec{T} + \vec{AG} \wedge \vec{P} + \vec{AE} \wedge \vec{Q} = \vec{0}$ (0,5) —— (2)

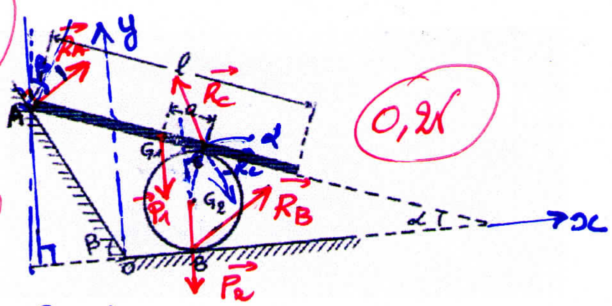
$\vec{AB}\begin{pmatrix}0\\ \frac{2}{3}a\\ 0\end{pmatrix}$ (0,5) ; $\vec{AC}\begin{pmatrix}a\\ a\\ 0\end{pmatrix}$ (0,5) ; $\vec{AG}\begin{pmatrix}a/2\\ a/2\\ 0\end{pmatrix}$ (0,5) ; $\vec{AE}\begin{pmatrix}a\\ \frac{2}{3}a\\ 0\end{pmatrix}$ (0,5)

(2) $\Leftrightarrow$ (0,75)
$$\begin{cases} \frac{2}{3}a R_{Bz} + aT\frac{\sqrt{3}}{2} - \frac{4aP}{3} - \frac{aP}{2} = 0 \\ -aT\frac{\sqrt{3}}{2} + 2aP + \frac{aP}{2} = 0 \\ -\frac{2}{3}a R_{Bx} = 0 \end{cases}$$

$$\left.\begin{array}{l} R_{Bx} = 0 \\ R_{Bz} = -P \end{array}\right\} \Rightarrow R_B = P \text{ (0,5)} ; \quad R_{Ax} = \frac{5\sqrt{6}}{12}P ; \; R_{Ay} = \frac{5\sqrt{6}}{12}P ; \; R_{Az} = \frac{3}{2}P.$$

$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2 + R_{Az}^2} = 17,39P$ (0,5)

**Exercice 3** : (5 pts)




(0,25)

* Equilibre de la plaque

$\sum \vec{F_i} = \vec{0} \Rightarrow \vec{P_1} + \vec{R_A} + \vec{R_c} = \vec{0}$ (0,25)

(0,25) $\vec{P_1}\begin{pmatrix}0\\ -m_1g\end{pmatrix}$ ; $\vec{R_A}\begin{pmatrix}\sin\beta\\ \cos\beta\end{pmatrix}R_A$ (0,25) ; $\vec{R_c}\begin{pmatrix}R_{cx}\\ R_{cy}\end{pmatrix}$ (0,25)

(0,5)
$$\begin{cases} R_A \sin\beta + R_{cx} = 0 & \text{—— (1)} \\ R_A\cos\beta + R_{cy} - m_1 g = 0 & \text{—— (2)} \end{cases}$$

* Equilibre du cylindre

$\sum \vec{F_i} = \vec{0} \Rightarrow \vec{P_2} + \vec{R_B} - \vec{R_c} = \vec{0}$ ; (0,25) $\vec{P_2}\begin{pmatrix}0\\ -m_2g\end{pmatrix}$ (0,25) ; $\vec{R_B}\begin{pmatrix}R_{Bx}\\ R_{By}\end{pmatrix}$ (0,25) ; $-\vec{R_c}\begin{pmatrix}-R_{cx}\\ -R_{cy}\end{pmatrix}$ (0,25)

(0,5)
$$\begin{cases} R_{Bx} - R_{cx} = 0 & \text{—— (3)} \\ R_{By} - R_{cy} - m_2 g = 0 & \text{—— (4)} \end{cases}$$

* $\sum \vec{M}_c(\vec{F_i}) = \vec{0} \Rightarrow \vec{CG_1} \wedge \vec{P_1} + \vec{CA} \wedge \vec{R_A} = \vec{0}$ (0,25) (équilibre de la plaque seule)

(0,25) $\vec{CG_1}\begin{pmatrix}-\cos\alpha\\ \sin\alpha\end{pmatrix}a$ ; $\vec{CA}\begin{pmatrix}-\cos\alpha\\ \sin\alpha\end{pmatrix}(a + \frac{L}{2})$ (0,25)

$\Rightarrow a\, m_1 g\cos\alpha - (a + \frac{L}{2}) R_A \cos(\alpha - \beta) = 0 \Rightarrow R_A = \dfrac{a\, m_1 g \cos\alpha}{(a+\frac{L}{2})\cos(\alpha-\beta)}$ (0,25)

$R_{Bx} = R_{Cx} = \dfrac{a\, m_1 g\cos\alpha \sin\beta}{(a+\frac{L}{2})\cos(\alpha-\beta)}$ (0,25)

$R_{cy} = m_1 g + \dfrac{a\, m_1 g\cos\alpha\cos\beta}{(a+\frac{L}{2})\cos(\alpha-\beta)}$ (0,25) ; $R_{By} = (m_1+m_2)g + \dfrac{a\, m_1 g\cos\alpha\cos\beta}{(\alpha+\frac{L}{2})\cos(\alpha-\beta)}$ (0,25)