Partie I

Solution Ex 1

$$S_{11} = E_1^{-1} = 5,780\,10^{-3}\text{GPa}^{-1},$$
$$S_{12} = S_{21} = -\nu_{12}E_1^{-1} = -0,208\,10^{-3}\text{GPa}^{-1},$$
$$S_{22} = E_2^{-1} = 30,211\,10^{-3}\text{GPa}^{-1},$$
$$S_{13} = S_{31} = -\nu_{13}E_1^{-1} = -1,445\,10^{-3}\text{GPa}^{-1},$$
$$S_{33} = E_3^{-1} = 193,424\,10^{-3}\text{GPa}^{-1},$$

$$S_{23} = S_{32} = -\nu_{23}E_2^{-1} = -5,166\,10^{-3}\text{GPa}^{-1},$$
$$S_{44} = E_4^{-1} = 308,642\,10^{-3}\text{GPa}^{-1},$$
$$S_{55} = E_5^{-1} = 120,919\,10^{-3}\text{GPa}^{-1},$$
$$S_{66} = E_6^{-1} = 106,610\,10^{-3}\text{GPa}^{-1},$$

Solution Ex 2

Matrice de passage

$$P=\begin{bmatrix}\cos 30 & \sin 30 & 0\\ -\sin 30 & \cos 30 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0.87 & 0.5 & 0\\ -0.5 & 0.87 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Matrice de transformation des contraintes

$$T_\sigma=\begin{bmatrix}0.75 & 0.25 & 0 & 0 & 0 & 0.87\\ 0.25 & 0.75 & 0 & 0 & 0 & -0.87\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.87 & -0.5 & 0\\ 0 & 0 & 0 & 0.5 & 0.87 & 0\\ -0.435 & 0.435 & 0 & 0 & 0 & 0.5\end{bmatrix}$$

$$T_\varepsilon=\left(T_\sigma^{-1}\right)^T$$

$$C'=T_\sigma.C.T_\sigma^T$$

$$S'=C'^{-1}$$

$$
\boldsymbol{C} = \begin{bmatrix} 173,415 & 1,425 & 1,334 & 0 & 0 & 0 \\ 1,425 & 33,271 & 0,899 & 0 & 0 & 0 \\ 1,334 & 0,899 & 5,205 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3,24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8,27 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9,38 \end{bmatrix} \text{GPa},
$$

$$
\boldsymbol{S} = \begin{bmatrix} 5,780 & -0,208 & -1,445 & 0 & 0 & 0 \\ -0,208 & 30,211 & -5,166 & 0 & 0 & 0 \\ -1,445 & -5,166 & 193,424 & 0 & 0 & 0 \\ 0 & 0 & 0 & 308,642 & 0 & 0 \\ 0 & 0 & 0 & 0 & 120,919 & 0 \\ 0 & 0 & 0 & 0 & 0 & 106,610 \end{bmatrix} 10^{-3}\text{GPa}^{-1}
$$

$$
\{\varepsilon\} = T_{\varepsilon}^{-1}\{\varepsilon'\}
$$

$$
\{\sigma\} = C\{\varepsilon\}
$$

$$
\{\sigma'\} = C'\{\varepsilon'\}
$$

Solution Ex 3

Matériau isotrope transverse avec : Plan d'isotropie 1-2

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

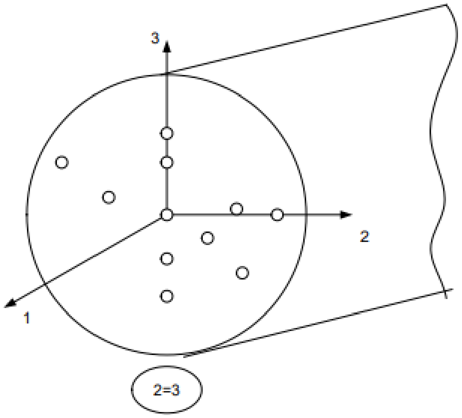
$$S_{11} = S_{22} \Rightarrow \frac{1}{E_1} = \frac{1}{E_2} \quad \text{donc} \quad E_1 = E_2$$

$$S_{13} = S_{23} \Rightarrow \frac{\nu_{31}}{E_3} = \frac{\nu_{32}}{E_3} \quad \text{donc} \quad \nu_{31} = \nu_{32}$$

$$S_{44} = S_{55} \Rightarrow \; G_{23} = G_{13}$$

$$S_{31} = S_{32} \Rightarrow \frac{\nu_{13}}{E_1} = \frac{\nu_{23}}{E_2} \quad \text{donc} \quad \nu_{13} = \nu_{23}$$

$$S_{66} = 2\left(S_{11} - S_{12}\right) \Rightarrow \frac{1}{G_{12}} = 2\left(\frac{1}{E_1} + \frac{\nu_{12}}{E_1}\right) \Rightarrow G_{12} = \frac{E_1}{2\left(1+\nu_{12}\right)}$$



$$G_{13} = G_{12}$$
$$E_2 = E_3$$
$$\nu_{21} = \nu_{31}$$
$$\nu_{23} = \nu_{32}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{21}/E_2 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{23}/E_2 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & -\nu_{23}/E_2 & 1/E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{12} & 0 \\ 0 & 0 & 0 & 0 & & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

Solution Ex 5

Tableau 2.3 ⇒ $\left\{\begin{array}{l} AS/3501 \\ \text{Graphite} \\ \text{Expoxy} \end{array}\right.$ $\left\{\begin{array}{l} E_1 = 138 \text{ GPa} \\ E_2 = 9 \text{ GPa} \\ G_{12} = 6.9 \text{ GPa} \\ \nu_{12} = 0.3 \end{array}\right.$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = \nu_{12}\frac{E_2}{E_1}$$

$$= 0.3 \times \frac{9}{138} = 0.0196$$

Matrice de rigidité $[Q]$:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{138}{1-0.3\times 0.0196} = 138.8 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.3\times 9}{1-0.3\times 0.0196} = 2.716 \text{ GPa} = Q_{21}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{9}{1-03\times 0.0196} = 9.05 \text{ GPa}$$

$$Q_{66} = G_{12} = 6.9 \text{ GPa}$$

Matrice de souplesse $[S]$:

$$S_{11} = \frac{1}{E_1} = \frac{1}{138} = 0.00725 \ (\text{GPa})^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.3}{138} = -0.00217 \ (\text{GPa})^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{9} = 0.111 \ (\text{GPa})^{-1}$$

TABLEAU 2.3 Constantes élastiques fondamentales des composites usuels [Gibson]

Composites	E_1 Msi (GPa)	E_2 Msi (GPa)	G_{12} Msi (GPa)	ν_{12}	v_f
T300/934 Graphite/epoxy	19.0 (131)	1.5(10.3)	1.0(6.9)	0.22	0.65
AS/3501 Graphite/epoxy	20.0(138)	1.3(9.0)	1.0(6.9)	0.3	0.65
p-100/ERL 1962 pitch graphite/expoxy	68.0(468.9)	0.9(6.2)	0.81(5.58)	0.31	0.62
Kevlar® 49/934 aramid/epoxy	11.0(75.8)	0.8(5.5)	0.33(2.3)	0.34	0.65
Scotchply® 1002 E-glass/epoxy	5.6(38.6)	1.2(8.27)	0.6(4.14)	0.26	0.45
Boron/5505 Boron/epoxy	29.6(204)	2.68(18.5)	0.81(5.59)	0.23	0.5
Spectra® 900/826 Polyethylene/epoxy	4.45(30.7)	0.51(3.52)	0.21(1.45)	0.32	0.65
E-glass/470-36 E-glass/vinylester	3.54(24.4)	1.0(6.87)	0.42(2.89)	0.32	0.30

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