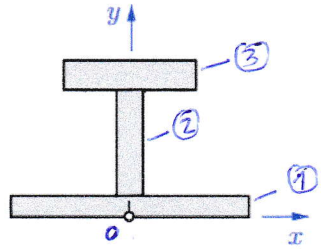
Ex.3 (1)

La structure est symétrique / $O\vec{y}$.

$x_G = 0$; $y_G = ?$

On divise la structure en 3 parties.

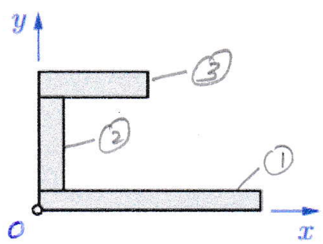


$$y_G = \frac{\sum y_i A_i}{\sum A_i}$$

$$= \frac{2\,(4\cdot 45) + 14(5\cdot 20) + 27\,(6\cdot 20)}{4\cdot 45 + 5\cdot 20 + 6\cdot 20}$$

$$= \frac{5000}{400} = \underline{\underline{12.5\ \text{mm}}}.$$

Ex.3 (2)



$$x_G = \frac{22.5\,(4\cdot 45) + 2.5\,(5\cdot 20) + 10\,(6\cdot 20)}{4\cdot 45 + 5\cdot 20 + 6\cdot 20}$$

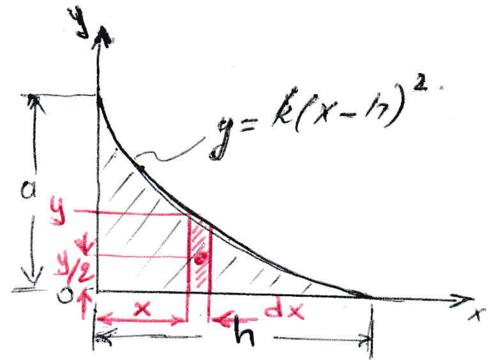
$$= \frac{5500}{400} = \underline{\underline{13.75\ \text{mm}}},$$

$$y_G = \frac{2\,(4\cdot 45) + 14\,(5\cdot 20) + 27\,(6\cdot 20)}{400}$$

$$= \underline{\underline{12.5\ \text{mm}}}.$$

Ex.4 (3)

$k = cte$



Considérons l'élément de surface ds.

$$ds = y\,dx \qquad 0 \leq x \leq h$$
$$y = k(x-h)^2$$

$$x_G = \frac{\int x\,ds}{\int ds}$$

$$\int ds = \int_0^h y\,dx = k\int_0^h (x-h)^2\,dx = k\int_0^h (x-h)^2\,d(x-h) = \frac{kh^3}{3}$$

$$\int x\,ds = k\int_0^h x\,(x-h)^2\,dx = k\int_0^h (x^3 - 2hx^2 + xh^2)\,dx = \frac{kh^4}{12}$$

$$x_G = \frac{h}{4}$$

$$y_G = \frac{\int \frac{y}{2}\,ds}{\int ds}$$

centre de l'élt de coordonnées $(x, y/2)$.

$$\int ds = \frac{kh^3}{3}$$

$$\int \frac{1}{2} y\,ds = \frac{k^2}{2}\int_0^h (x-h)^4\,dx$$

$$= \frac{k^2}{2}\int_0^h (x-h)^4\,d(x-h) = \frac{1}{10}k^2h^5$$

$$y_G = \frac{3}{10}kh^2$$