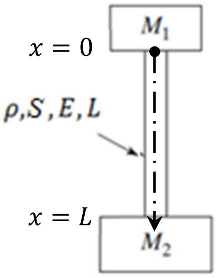
Exercice 3

b)



$$u(x,t) = U(x).T(t)$$

$$T(t) = A\sin\omega t + B\cos\omega t \Rightarrow \frac{d^2T(t)}{dt^2} = -\omega^2 T(t)$$

$$U(x) = C\sin\omega\sqrt{\frac{\rho}{E}}x + D\cos\omega\sqrt{\frac{\rho}{E}}x \Rightarrow$$

$$\frac{dU(x)}{dx} = C\omega\sqrt{\frac{\rho}{E}}\cos\omega\sqrt{\frac{\rho}{E}}x - D\omega\sqrt{\frac{\rho}{E}}\sin\omega\sqrt{\frac{\rho}{E}}x$$

<u>Conditions aux limites</u> :

$x = 0$	$ES\frac{\partial u}{\partial x}(0,t) = M_1\frac{\partial^2 u}{\partial t^2}(0,t)$
$x = L$	$ES\frac{\partial u}{\partial x}(L,t) = -M_2\frac{\partial^2 u}{\partial t^2}(L,t)$

■ $ES\frac{\partial u}{\partial x}(0,t) = M_1\frac{\partial^2 u}{\partial t^2}(0,t) \Rightarrow ES\,T(t)\frac{dU}{dx}(0) = M_1\frac{d^2T(t)}{dt^2}U(0) = -\omega^2 M_1 T(t)U(0)$

$$\Rightarrow ES\frac{dU}{dx}(0) = -\omega^2 M_1 U(0)$$

$$\Rightarrow ES\,C\omega\sqrt{\frac{\rho}{E}} = -\omega^2 M_1 D \Rightarrow C = \frac{-\omega M_1}{ES\sqrt{\frac{\rho}{E}}}D \qquad \mathbf{(1)}$$

■ $ES\frac{\partial u}{\partial x}(L,t) = -M_2\frac{\partial^2 u}{\partial t^2}(L,t) \Rightarrow ES\frac{dU}{dx}(L)T(t) = -M_2\frac{d^2T(t)}{dt^2}U(L) = \omega^2 T(t)M_2 U(L)$

$$\Rightarrow ES\frac{dU}{dx}(L) = \omega^2 M_2 U(L)$$

$$\Rightarrow ES\left(C\omega\sqrt{\frac{\rho}{E}}\cos\omega\sqrt{\frac{\rho}{E}}L - D\omega\sqrt{\frac{\rho}{E}}\sin\omega\sqrt{\frac{\rho}{E}}L\right) = \omega^2 M_2\left(C\sin\omega\sqrt{\frac{\rho}{E}}L + D\cos\omega\sqrt{\frac{\rho}{E}}L\right) \quad \mathbf{(2)}$$

En substituant (1) dans (2) et en simplifiant par D, on obtient :

$$-\omega^2 M_1\cos\omega\sqrt{\frac{\rho}{E}}L - ES\omega\sqrt{\frac{\rho}{E}}\sin\omega\sqrt{\frac{\rho}{E}}L = \frac{-\omega^3 M_1 M_2}{ES\sqrt{\frac{\rho}{E}}}\sin\omega\sqrt{\frac{\rho}{E}}L + \omega^2 M_2\cos\omega\sqrt{\frac{\rho}{E}}L$$

$$\Rightarrow \omega^2(M_1+M_2)\cos\omega\sqrt{\frac{\rho}{E}}L = \omega\sqrt{\frac{\rho}{E}}\left(\frac{\omega^2 M_1 M_2}{\rho S} - ES\right)\sin\omega\sqrt{\frac{\rho}{E}}L$$

D'où :

$$\tan\omega\sqrt{\frac{\rho}{E}}L = \frac{\omega(M_1+M_2)}{\sqrt{\frac{\rho}{E}}\left(\frac{\omega^2 M_1 M_2}{\rho S} - ES\right)}$$

En posant :

$$X = \omega\sqrt{\frac{\rho}{E}}L$$

On aura :

$$\tan X = \frac{X(M_1+M_2)}{\frac{X^2 M_1 M_2}{\rho SL} - \rho SL}$$

En désignant la masse de la barre par M_b :

$$M_b = \rho SL$$

On obtient :

$$\tan X = \frac{X(M_1+M_2)}{\frac{X^2 M_1 M_2}{M_b} - M_b}$$

Soit :

$$\tan X = \frac{X(M_1+M_2)M_b}{X^2 M_1 M_2 - {M_b}^2}$$

Ou :

$$\tan X = \frac{X\left(\frac{M_b}{M_1} + \frac{M_b}{M_2}\right)}{X^2 - \frac{M_b}{M_1}.\frac{M_b}{M_2}}$$