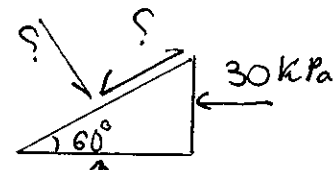
Corrigé de la série : Contraintes dans les sols.

Exercice n° 01 :



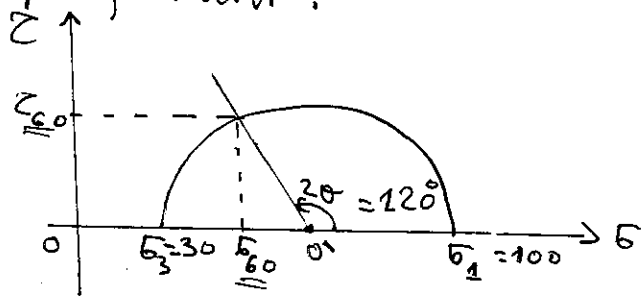
* Analytiquement :

Pour déterminer σ_θ et τ_θ, on utilise les relations :

$$\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_3 - \sigma_1}{2}\cos 2\theta \quad \text{et} \quad \tau_\theta = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta.$$

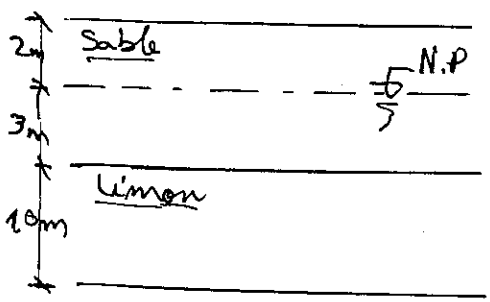
$$\Rightarrow \boxed{\sigma_\theta = 47,4 \text{ kPa}} \quad \text{et} \quad \boxed{\tau_\theta = 30,2 \text{ kPa}}.$$

* Graphiquement :

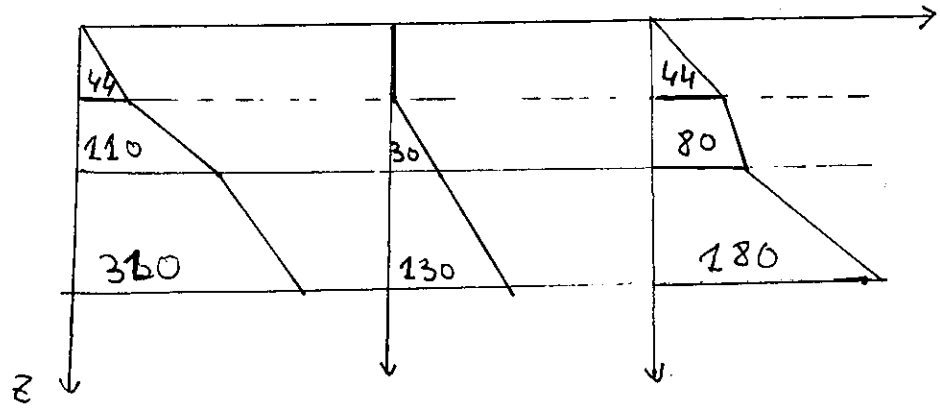


$$\begin{cases} R = \dfrac{\sigma_1 - \sigma_3}{2} \\ OO' = \dfrac{\sigma_1 + \sigma_3}{2} \end{cases}$$

Exercice n° 03



σ_v − u = σ_v'



Par définition :

- la contrainte verticale totale : $\sigma_v = \gamma . z$.
- la pression interstitielle : $u = \gamma_w . z$.
- la contrainte effective verticale : $\sigma_v' = \gamma' . z = \sigma_v - u$.

$0 \leq z \leq 2\text{m}$

* $\sigma_v = \gamma_{sable} \times z = \begin{cases} z = 0 \rightarrow \sigma_v = 0 \text{ kN/m}^2. \\ z = 2\text{m} \rightarrow \sigma_v = 22 \times 2 = 44 \text{ kN/m}^2. \end{cases}$

* $u = 0$ (la nappe d'eau commence à partir de -2.00 m)

* $\sigma'_v = \sigma_v = \begin{cases} z=0 \rightarrow \sigma'_v = 0 \\ z=2m \rightarrow \sigma'_v = 44\,KN/m^2 \end{cases}$

$2 \leq z \leq 5\,m$

* $\sigma_v = \gamma_{sable} \times 2 + \gamma_{sable} \times z = \begin{cases} z=0 \rightarrow \sigma_v = 44\,KN/m^2 \\ z=3m \rightarrow \sigma_v = 22\times2 + 22\times3 = 110\,KN/m^2 \end{cases}$

* $U = \gamma_w \cdot z = \begin{cases} z=0 \rightarrow U = 0 \\ z=3 \rightarrow U = 10\times3 = 30\,KN/m^2 \end{cases}$

* $\sigma'_v = \gamma' \cdot z = (\gamma_{sat} - \gamma_w) z + \gamma_{sable}\times2 = 12.z + 22\times2 = \begin{cases} z=0 \rightarrow \sigma'_v = 44\,KN/m^2 \\ z=3m \rightarrow \sigma'_v = 80\,KN/m^2 \end{cases}$

$5 \leq z \leq 15\,m$.

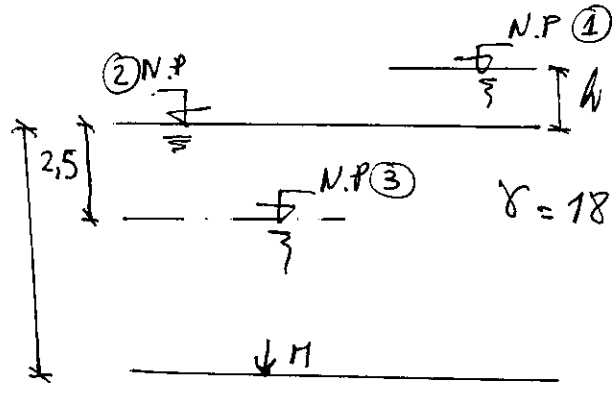
* $\sigma_v = \gamma_{sable} \times 5 + \gamma_{lim} \times z \begin{cases} z=0 \rightarrow \sigma_v = 22\times5 + 0 = 110\,KN/m^2 \\ z=10 \rightarrow \sigma_v = 22\times5 + 20\times10 = 310\,KN/m^2 \end{cases}$

* $U = 10\times3 + 10\times z \begin{cases} z=0 \rightarrow = 30\,KN/m^2 \\ z=10 \rightarrow U = 130\,KN/m^2 \end{cases}$

* $\sigma'_v = \sigma_v - U$
$= \gamma_S \times 2 + \gamma'_S \times 3 + \gamma'_L \times z = 22\times2 + 12\times3 + 10\times z = \begin{cases} z=0 \rightarrow \sigma'_v = 80\,KN/m^2 \\ z=10m \rightarrow \sigma'_v = 180\,KN/m^2 \end{cases}$

Exercice n° 04

σ_v, σ'_v, U ?



1\ La nappe à une cote "h" au dessus du terrain naturel :

* $\sigma_v = \gamma_w \cdot h + \gamma_{sat} \times 5 = 10h + 18\times5 = 10h + 90$.
* $U = \gamma_w (h + 5)$.
* $\sigma'_v = \sigma_v - U = (\gamma_w h + \gamma_{sat} \times 5) - (\gamma_w h + \gamma_w \times 5)$
$= (\gamma_{sat} - \gamma_w) 5 = (18 - 10) 5 = 40\,KN/m^2$.

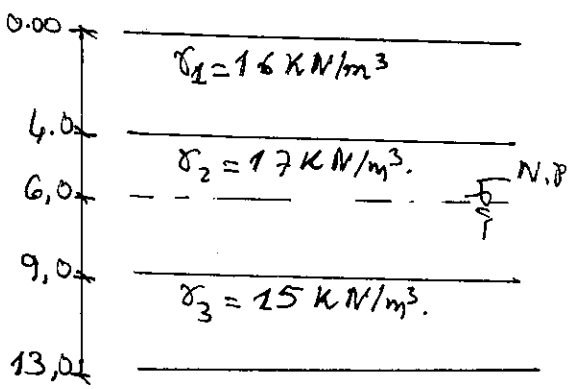
2°\ En surface ($z = 5$ m).

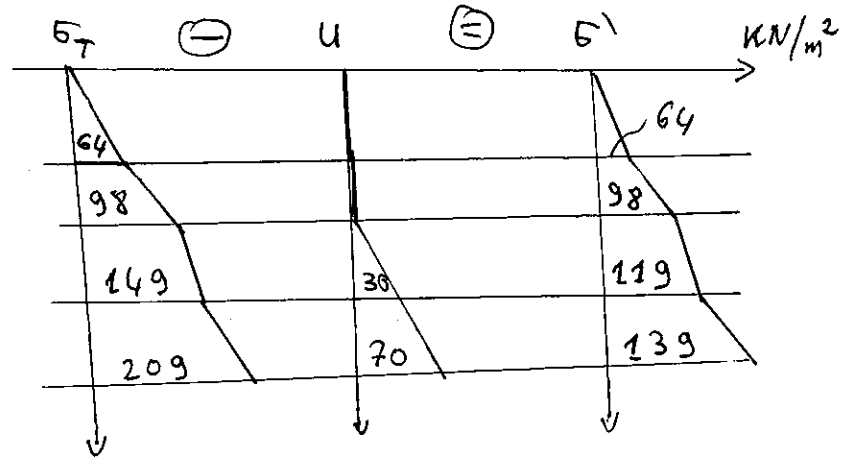
* $\sigma_v(H) = \gamma_{sat} \cdot z = 18 \times 5 = 90\ KN/m^2$.
* $U = \gamma_w \cdot z = 10 \times 5 = 50\ KN/m^2$.
* $\sigma_v' = \sigma_v - U = \gamma' \cdot z = 8 \times 5 = 40\ KN/m^2$.

3°\ à 2,5 m de profondeur.

* $\sigma_v = \gamma_h \times 2,5 + \gamma_{sat} \times 2,5 = 90\ KN/m^2$ ($\gamma_h = \gamma_{sat}$).
* $U = \gamma_w \cdot z_w = 10 \times 2,5 = 25\ KN/m^2$.
* $\sigma_v' = \sigma_v - U = \gamma_h \times 2,5 + (\gamma_{sat} \times 2,5 - \gamma_w \times 2,5) = 65\ KN/m^2$.

Exercice n° 05





$0 \leq z \leq 4$ m

* $\sigma_T = \gamma_1 \cdot z = \begin{cases} z=0 \rightarrow \sigma_T = 0 \\ z=4 \rightarrow \sigma_T = 64\ KN/m^2. \end{cases}$
* $U = 0$
* $\sigma' = \sigma_T - U = \begin{cases} z=0 \rightarrow \sigma' = 0 \\ z=4 \rightarrow \sigma' = 64\ KN/m^2. \end{cases}$

$4\text{ m} \leq z \leq 6,0$ m.

* $\sigma_T = \gamma_1 \times h_1 + \gamma_2 \cdot z = 64 * 17 \times z = \begin{cases} z=0 \rightarrow \sigma_T = 64\ KN/m^2. \\ z=2 \rightarrow \sigma_T = 98\ KN/m^2. \end{cases}$
* $U = 0$
* $\sigma' = \sigma_T$.

$6\text{ m} \leq z \leq 9$ m.

* $\sigma_T = \gamma_1 \cdot h_1 + \gamma_2 \times h_2 + \gamma_{2sat} \times z = 16 \times 4 + 17 \times 2 + 17 \times z$

$= 98 + 17z = \begin{cases} z=0 \rightarrow \sigma_T = 98\ KN/m^2 \\ z=3m \rightarrow \sigma_T = 149\ KN/m^2. \end{cases}$

* $U = \gamma_w \cdot z_w = 10 \times z = \begin{cases} z=0 \rightarrow U=0 \\ z=3m \rightarrow U = 30\ KN/m^2. \end{cases}$

* $\sigma' = \sigma_T - U = \gamma_1 h_1 + \gamma_2 h_2 + \gamma'_2 z = 98 + 7z = \begin{cases} z=0 \rightarrow \sigma' = 98\ KN/m^2 \\ z=3m \rightarrow \sigma' = 119\ KN/m^2 \end{cases}$

$9 \leq z \leq 13\ m$

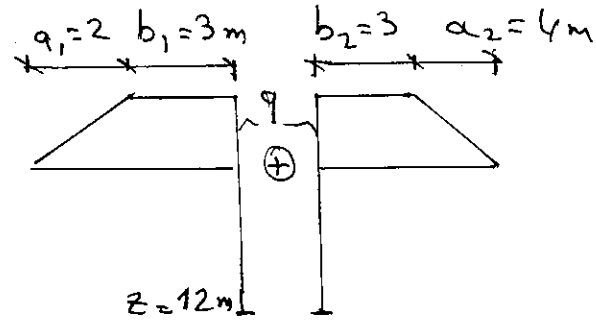
* $\sigma_T = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_{sat_2} h_{r_3} + \gamma_3 \cdot z.$

$= \underbrace{16 \times 4 + 17 \times 2 + 17 \times 3}_{\sigma_T (9m)} + 15 \cdot z = 149 + 15 \times z = \begin{cases} z=9 \rightarrow \sigma_T = 149\ KN/m^2 \\ z=4m \rightarrow \sigma_T = 209\ KN/m^2. \end{cases}$

* $U = U_{(9m)} + \gamma_w \cdot z = 30 + 10z = \begin{cases} z=0 \rightarrow U = 30\ KN/m^2. \\ z=4m \rightarrow U = 70 KN/m^2. \end{cases}$

* $\sigma' = \sigma_T - U = \sigma'_{(9m)} + \gamma'_3 \cdot z = 119 + 5 \cdot z = \begin{cases} z=0 \rightarrow \sigma' = 119 KN/m^2 \\ z=4m \rightarrow \sigma' = 139\ KN/m^2. \end{cases}$

Exercice n° 06



$q_{remblai} = \gamma \times z = 20 \times 5$

$q_{remb.} = 100\ KN/m^2.$

$\Delta\sigma_z = \Delta\sigma_{z_1} + \Delta\sigma_{z_2} = (I_1 + I_2)\, q.$

* $I_1 = f\left(\frac{a_1}{z} ; \frac{b_1}{z}\right) = f\left(\frac{2}{12} ; \frac{3}{12}\right) = 0,2.$

$\Rightarrow \Delta\sigma_{z_1} = 100 \times 0,2 = \underline{20\ KN/m^2}.$

* $I_2 = f\left(\frac{a_2}{z}, \frac{b_2}{z}\right) = f\left(\frac{4}{12} ; \frac{3}{12}\right) = f(0,33 ; 0,25) = 0,235$

$\Rightarrow \Delta\sigma_{z_2} = \underline{23,5\ KN/m^2}$

d'où : $\Delta\sigma_z = 20 + 23,5 = 43,5\ KN/m^2.$