Tutorial 03 : Curvilinear integrals

Exercise 01

1) Evaluate the following line integrals : $\int_C F(x,y)dx$, $\int_C F(x,y)dy$, $\int_C F(x,y)dl$.

Where $F(x,y) = 2x + y^2$, And (C) is the line $y = 2x$ From $(0,0)$ to $(1,2)$.

2) Evaluate the line integrals : $\int_C (4xy - 3x^2z^2)dl$ Where $(C)$is the segment $[AB]$. $A(0,0,0), B(1,1,1)$.

Exercise 02

1) Find the line integral $\int_C (2y + x^2)dx + 3xydy$ Where (C) represents :

i) The segment $[AB]$, $A(0,3), B(2,4)$.

ii) The path joining the line segment : From $A(0,3)$ to $C(2,3)$, and from $C(2,3)$ to $B(2,4)$.

iii) The parabola of equation : $y = \frac{x^2}{4} + 3$ From $A(0,3)$ to $B(2,4)$.

2) Evaluate the line integral $\int_C yx^2dx + 4xy^2dy - 6xzdz$. Where (C) : $\begin{cases} x(t) = 2t \\ y(t) = t^2 \\ z(t) = t^3 \end{cases}$

From $(0,0,0)$ to $(4,1,1)$.

Exercise 03

If $F(x,y,z) = y\vec{\imath} + 3x\vec{\jmath}$. Evaluate the circulation of F along the circle

of the center $(0,0)$, and the radius 2 oriented counterclockwise.

Exercise 04

Find the área of the region in the second quadrant bounded by the ellipse $4x^2 + 9y^2 = 36$.

Exercise 05

1) Show that $I = \int_{(0,0)}^{(2,1)} (2xy + 1)dx + (x^2 - 2y)dy$ is indepandent of the path taken.

2) Find I by tow differents methods.

3) Find $\int_{(C)} (2xy + 1)dx + (x^2 - 2y)dy$, Where (C) is :

(a) The circle $x^2 + y^2 = 1$.

(b) The square with vertices $(0,0), (1,0), (1,1), (0,1)$.